Comité "Emploi - travail - redistribution"

Modèles collectifs d'offre de travail

Responsable scientifique : P.-A. Chiappori *

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L'objectif poursuivi dans cette recherche consistait à développer et étendre la formalisation collective des comportements d'offre de travail des ménages. Rappelons que, dans la formalisation "unitaire", un ménage est assimilé à un décideur unique; en particulier, son offre de travail est analysée comme résultant de la maximisation d'une fonction d'utilité censée refléter les préférences "d'ensemble" du ménage. Le problème, évidemment, est que cette représentation n'a pas de fondement réellement solide. On sait, depuis Arrow (voire Condorcet), qu'il est en général impossible de définir une utilité représentative d'un groupe d'agents, sauf à accepter des hypothèses très fortes (existence d'un dictateur, par exemple); et les analyses plus poussées de la question confirment ce pessimisme.

Cependant, si cette critique est souvent acceptée, il n'existait, jusqu'à récemment, aucun modèle explicite et convainquant de comportement qui se distingue réellement de la forme "unitaire". Un premier pas a été effectué dans deux articles (Chiappori 1988,1992) consacrés à un modèle élémentaire d'offre de travail. On montrait, dans ce cadre, deux résultats intéressants:

. d'une part, le modèle collectif pouvait engendrer des *restrictions testables* sur les comportements observés.

. d'autre part, il était possible, à partir de fonctions d'offre satisfaisant les restrictions, d'identifier les caractéristiques structurelles (préférences, règle de décision) sous-jacentes.

Ces résultats étaient particulièrement stimulants, dans la mesure où ils suggéraient que la direction de recherche retenue ouvrait bien des perspectives prometteuses. Cependant, le modèle de base souffrait d'une simplification importante des hypothèses. Citons quelques lacunes évidentes : caractère continu des offres de travail; absence de consommation collective; absence de production domestique; formulation statique; etc. L'objectif du projet était donc de contribuer à pallier quelques-unes de ces imperfections.

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1. Travail marchand et travail domestique

Un premier progrès a consisté à prendre en compte, dans le cadre du modèle initial, l'existence d'un travail domestique en sus du travail marchand. Une première version montrait que les résultats du modèle de base s'étendaient à la condition que le *prix* du bien domestique s'égalise entre les ménages (ce qui est le cas, par exemple, s'il existe un marché des services de ce type); notons que l'on ne requiert pas, en revanche, d'observer ce prix, ni d'ailleurs la *production* de bien domestique à l'intérieur du ménage. Des développements récents ont conduit à un réexamen du problème, allant dans le sens d'une plus grande généralité. En particulier, on a étudié en détail le cas où le prix du bien n'était pas uniforme entre ménages, soit que le marché correspondant n'existe pas, soit que des solutions de coin (non participation, par exemple) soient obtenues.

Dans ce cadre, deux résultats ont été prouvés :

. d'une part, si l'on suppose l'existence d'au moins deux sources de revenu non salarial (par exemple du fait de revenus spécifiques du mari et de la femme), alors l'impact de chaque source sur la règle de partage peut être exactement identifié, même si l'effet des salaires reste ambigu. Cette propriété peut se révéler précieuse, notamment lorsque l'analyse a pour but d'estimer les effets d'aides spécifiques (allocations,...), qui par définition interviennent comme des revenus non salariaux, et qui, de plus, sont souvent versées formellement à l'un des conjoints (la mère en général).

. par ailleurs, on montre que des hypothèses plus spécifiques sur la forme des processus de décision permettent de lever l'indétermination. Par exemple, si l'on suppose que la négociation intrafamiliale ne dépend que des contributions respectives des membres au revenu total, alors la règle de partage peut être identifiée (toujours à une constante près).

Ce travail a donné lieu à publication dans le *Journal of Political Economy*; on trouvera, en Annexe 1, une copie de cet article.

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2. Offre de travail discrète et participation

Seconde extension, de nature plus empirique : l'extension au cas d'une offre de travail discrète. Dans la formalisation unitaire, une bonne part de l'économétrie de l'offre de travail se fait à partir de modèles dits de participation, dans lesquels on étudie la décision d'entrée sur le marché du travail. La théorie de ces modèles a été faite il y a plusieurs années; on sait, notamment, qu'il est possible d'identifier dans un tel contexte la fonction d'utilité sous-jacente. La contrepartie "collective" de ces travaux fait l'objet d'une collaboration avec T. Magnac (INRA) et les Professeurs R. Blundell et C. Meghir (University College, Londres). Dans ce cadre, le financement obtenu a permis de nombreux échanges entre participants, et notamment plusieurs séjours de longue durée de chercheurs français ou anglais auprès de l'équipe partenaire.

Diverses avancées ont été réalisées. Tout d'abord, un examen préliminaire des données a été effectué (il s'agit de données britanniques). On a pu constater une dissymétrie nette selon le sexe. L'offre de travail des hommes est essentiellement discrète, et se ramène à un choix activité à temps complet et inactivité. En revanche, le nombre d'heures de travail des femmes est beaucoup plus lisse, et varie de façon quasi continu (même si, évidemment, des modes sont perceptibles autour des durées de mi-temps et plein-temps).

Ces faits stylisés ont été traduits dans un modèle où l'offre de travail de l'un des membres est discrète, l'autre continue (avec cependant solution de coin pour la non-participation). La résolution théorique est terminée; on a montré, en particulier, qu'il était encore possible d'identifier les préférences et la règle de partage, bien que l'identification repose à présent sur des techniques beaucoup plus complexes.

La spécification économétrique a posé des problèmes beaucoup plus délicats. Le point central est l'introduction de facteurs d'hétérogénéité non observée. On a pu, en fait, introduire une structure assez riche, en autorisant trois niveaux d'hétérogénéité (affectant respectivement les préférences de l'homme, celles de la femme, et la règle de partage des revenus au sein du couple).

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Cette ambition est d'autant plus forte que le modèle se heurte par ailleurs aux problèmes classiques de l'économétrie de l'offre de travail discrète. Par exemple, la décision d'entrée sur le marché du travail est modélisée en termes de salaire de réserve; l'idée étant qu'un agent acceptera de participer si le salaire qui lui est offert dépasse un certain seuil, propre à l'agent. D'une part, ce seuil est inobservable, et doit être estimé. D'autre part, et plus grave, dans le cas des agents inactifs, le salaire potentiel est également non observé. La solution simple, consistant à recalculer un salaire 'moyen' à partir des caractéristiques propres de l'agent (âge, éducation, ...) est incorrecte en présence d'hétérogénéité; en effet, le fait qu'un agent soit inactif suggère que le salaire qui lui a été offert était sans doute inférieur à la moyenne ainsi calculée (techniquement, on a biais de sélection).

Le résultat fort, dans ce contexte, est que le modèle collectif ainsi construit est identifiable sous certaines hypothèses. De plus, celles-ci ne sont pas plus contraignantes que dans le cadre d'un modèle unitaire. Il s'agit, pour l'essentiel, de relations d'exclusion, impliquant l'existence de variables influençant certains aspects des phénomènes et pas certains autres. Il faut, par exemple, disposer, pour chaque membre, d'une variable affectant son salaire potentiel mais pas ses préférences (on peut penser à la situation du marché du travail), et, inversement, ses préférences mais pas son salaire potentiel (par exemple, son patrimoine). Tout ceci est a priori nécessaire pour l'estimation d'un modèle unitaire. La seule différence est la nécessité, dans le cadre collectif, de disposer également d'un *facteur de distribution*, c'est à dire d'une variable indépendante des préférences des agents qui puisse affecter le processus de décision intra-familial. Nous reviendrons plus loin sur ce point.

On trouvera, en Annexe 2, l'état actuel d'avancement du modèle théorique. En particulier, la note en question, de nature interne, reprend les équations d'identification, et montre par ailleurs comment l'estimation est possible.

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3. Offre de travail et 'marché du mariage'

Les travaux précédents partaient d'une modélisation discrète de l'offre de travail. Parallèlement, nous avons étudié une modélisation continue de ce même phénomène. L'idée est ici d'étudier le nombre total d'heures de travail par an, en fonction des déterminations habituelles (salaires, revenus non salariaux, déterminants socio-démographiques, etc.). Ce genre d'approche paraît plus pertinente dans le cadre nord américain, où le marché du travail est plus fluide, et où le choix du temps de travail (heures par semaine, mais aussi semaines par an et années dans le cycle de vie) beaucoup plus flexible qu'en Europe. Les applications empiriques ont donc porté sur des données issues du PSID (panel study of income dynamics) mis en place aux Etats-Unis, relatives à l'année 1988. Le travail a été mené en collaboration avec B. Fortin et G. Lacroix, de l'Université Laval à Québec; le financement reçu a notamment permis à ces deux spécialistes de séjourner à Paris pendant deux mois.

L'article correspondant, encore sous forme préliminaire, figure en annexe 3. On se contentera ici d'en souligner deux aspects.

. en utilisant les techniques décrites dans Chiappori (1992), généralisées pour l'occasion, on arrive à estimer les caractéristiques des processus de décision interne des ménages, et notamment la "règle de partage", qui décrit les transferts intra familiaux résultant de modifications de l'environnement. Les valeurs trouvées sont vraisemblables, mais suggèrent des effets de type 'pouvoirs de négociation' assez prononcés. Par exemple, on estime l'effet, sur les transferts internes, d'une hausse de 1$ du salaire horaire (soit, en termes annuels, un gain d'environ 2.000$). Les effets sur l'offre de travail suggèrent qu'en fait, la quasi-totalité du surplus reste à l'agent bénéficiaire de la hausse de salaire; pire, celui-ci s'avère même capable d'attirer une part plus importante du revenu non salarial.

. un second objectif du travail consistait à introduire, dans ces estimations, des 'facteurs de distribution', c'est à dire des variables qui influencent le processus de décision sans être endogènes par rapport aux préférences. Dans une ligne beckerienne, nous avons tenté d'utiliser

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le ratio hommes/femmes dans le comté en question. L'idée sous-jacente était que, dans un contexte où les femmes sont relativement 'rares', elles pourraient avoir une position meilleure dans une éventuelle négociation intra-familiale, notamment si cette rareté est corrélé avec une plus grande facilité de rencontre d'un partenaire de remplacement. Surtout, il s'agissait là d'une variable exogène - alors que les études précédentes, à commencer par les travaux relatifs à l'agrégation du revenu ("income pooling") risquaient d'être affectées par des biais d'endogénéité.

A notre grande surprise, la variable en question est sortie de façon significative des régression. De plus, son effet va exactement dans le sens prévu : un ratio favorable aux femmes (i.e., plus d'hommes) tend, toutes choses égales par ailleurs, à décroître l'offre de travail féminine et à accroître l'offre de travail masculine, ce qui, pour des effets revenus de sens habituel, suggère bien un transfert de revenu non salarial plus important à l'avantage des femmes. Quantitativement, l'effet est faible mais significatif : un peu moins de 300$ par point de ratio.

Une explication différente, mais assez naturelle, consisterait à supposer des effets sur le marché du travail : la rareté relative provoque (ou s'explique par) des différences de productivité entre hommes et femmes, compte tenu du type d'activités les plus développées localement. Cependant, cette explication aurait une conséquence testable : les effets seraient indépendants du statut marital, et devraient donc affecter aussi les célibataires. Or des régressions effectuées sur des célibataires ne montrent aucun effet de ce type.

Ajoutons également qu'une collaboration prometteuse a été entamée avec des équipes de recherche de l'Université chinoise de Hong Kong. Le soutien du CGP a permis le financement d'un séjour de deux semaines dans ce cadre. J'ai pu notamment travailler avec le professeur Zhang sur un modèle d'offre de travail collective où le loisir est en partie public. Les travaux de Zhang et Fong montrent que le modèle reste identifiable, à condition que l'on puisse également observer les demandes (en fonction des salaires respectifs) d'un bien assignable. L'application empirique portera sur des données de Chine continentale, dont dispose l'équipe de Hong Kong, et qui n'ont jamais été utilisées jusqu'ici. L'application de ces techniques à un pays en voie de développement est particulièrement prometteuse.

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4. Cadre général : offre de travail et demande de biens

Une troisième voie d'attaque a consisté à replacer le problème dans un cadre général : comment caractériser, de façon théorique et empirique, les propriétés d'une fonction générale de demande de biens et d'offre de travail issue d'un modèle collectif. Une première contribution en cours, en collaboration avec M. Browning, a porté sur la généralisation des conditions de Slutsky. On notera que la partie théorique est générale, et s'applique aussi bien aux fonctions de demande qu'à l'offre de travail. Par ailleurs, elle ne requiert aucune hypothèse sur la forme des préférences ou des consommations. En revanche, les test empiriques ne sont pour l'instant effectués que sur des fonctions de demande. En effet, l'une des conclusions de l'approche théorique est que, pour le contexte le plus général (et donc sans supposer, comme dans les travaux précédents, un caractère privé des consommations), cinq demandes au moins sont nécessaires. Il est donc nécessaire d'effectuer les tests à la fois sur des offres de travail et des demandes de biens; et l'on se heurte à l'absence de données adéquates. La dernière version de l'article est en Annexe 4. L'article a été accepté pour publication dans *Econometrica.*

Enfin, une recherche en cours, en collaboration avec le Professeur I. Ekeland, de l'Université Paris IX, porte sur les fondements mathématiques du modèle. Des résultats prometteurs ont été obtenus. On a montré, en particulier, que le problème relevait d'une approche générale classique en topologie différentielle (et plus précisément en calcul sur les variétés), faisant appel à la notion de calcul différentiel extérieur. Cette approche permet de généraliser les tests existant; par ailleurs, elle conduit à repenser de façon féconde le problème de l'identification. L'important est que l'on recherche, par ces techniques complexes, un résultat (fort) d'identification *non paramétrique*. L'intérêt de cette exigence est clair : si l'identifiabilité est prouvée, on peut alors choisir n'importe quelle forme paramétrique ou semi-paramétrique, en étant certain qu'aucune autre forme fonctionnelle n'aurait pu donner la même forme réduite pour les offres de travail. Il s'agit donc d'une exigence de robustesse qui se trouvera automatiquement satisfaite.

En fait, nous sommes parvenus récemment à une caractérisation complète des conditions d'identification. Celles-ci sont relativement restrictives; en général, des degrés de liberté

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demeurent, la règle de partage étant notamment estimée seulement modulo une fonction homogène des prix. Une solution possible, suggérée par J. Heckman, est d'estimer indépendamment les préférences individuelles, par exemple à partir de données sur les ménages mono-individuels. Dans ce cas, on identifie exactement l'ensemble du modèle. Enfin, les travaux entrepris dans ce cadre ont eu des sous-produits inattendus mais intéressants. Ainsi, la proximité entre ce problème et les questions d'agrégation de demandes individuelles, étudiées notamment par Sonnenschein, Debreu, Mas Colell et beaucoup d'autres, nous ont conduits à étudier un problème resté ouvert, celui de la décomposition de demandes de marché. Les techniques employées ont fait la preuve de leur efficacité en nous permettant de résoudre ce problème, resté sans solution depuis vingt ans. On trouvera en Annexe 5 l'article correspondant, qui est à paraître dans *Econometrica*.

Signalons pour conclure que le financement obtenu a permis le séjour en France, du premier juillet au 31 décembre 1995, du Professeur M. Browning, de l'Université McMaster (Canada), avec lequel une bonne part de la recherche passée a été menée. De nombreuses réflexions ont été entamées à cette occasion. A terme, on se propose notamment de reformuler le problème classique d'estimation des échelles d'équivalence, notamment selon la méthode due à Rothbard, en introduisant les idées développées dans le cadre de l'analyse collective.

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ANNEXE 1

Travail marchand et travail domestique

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Introducing Household Production in Collective Models of Labor Supply

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I. Introduction

In a recent paper (Chiappori 1992), I proposed a model of labor supply based on a "collective" representation of household behavior. In this framework, each household member is characterized by his or her own utility function, and decisions are only assumed to result in Pareto-efficient outcomes. I showed that these simple assumptions were sufficient to (i) generate testable restrictions on labor supply functions and (ii) recover individual preferences and the outcome of the decision process from observed behavior. For the sake of simplicity, the results were derived in the simplest possible case, namely, a static labor supply model with private consumptions. Such a framework was appropriate only for the purpose of the paper, that is, a preliminary exploration of the properties of collective models of labor supply. The next step is to evaluate the robustness of the results to various extensions of the basic model. This opens a general research program and generates a wide range of particular problems that it is hoped will be considered in the future.

Among the most serious shortcomings of the 1992 model is the absence of household production: as in most labor supply models, agents were assumed to divide their time between market activities and leisure. As Apps and Rees (1997, in this issue) very rightly point out in their comment, the absence of domestic production is far from innocuous as soon as welfare issues are considered. For instance, a low level of market labor supply will automatically be interpreted as a large consumption of leisure, whereas it may in fact reflect the specialization of one of the members in domestic

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production. This remark clearly suggests that the estimation of individual utilities in a collective labor supply model is likely to be very sensitive to the presence of household production. Also, Apps and Rees raise a major issue, namely, To what extent is it possible to recover the intrahousehold decision process in the presence of household production? They argue that the answer is negative in general, which suggests that the introduction of household production within the collective framework may lead to serious identification problems.

In the present note, I make four basic points. (1) A key issue is whether the commodity that is domestically produced is marketable (the "complete market" case) or not (the "incomplete market" framework). In their paper, Apps and Rees consider only the latter situation. I briefly discuss under which circumstances each setting can be seen as appropriate. (2) In the complete market case, I show that the results of the initial framework can be extended to household production. That is, testable restrictions on domestic and market labor supply functions can be derived, and the intrahousehold distribution of resources (the "sharing rule") can be recovered up to an additive constant. (3) In the alternative situation (the one Apps and Rees implicitly consider), the price of the household commodity is endogenous to household decisions and has to be estimated as well (as a function of wages and incomes). Then their result on the impossibility of exact identification is correct. It can actually be made more precise; specifically, I show that if the household production exhibits constant returns to scale, the sharing rule can be recovered only *up to an additive function of wages*. In particular, I show, in a counterexample, why parametric identification can then be revealed as seriously misleading. (4) However, complete identification (up to an additive constant) may still be possible, but it requires additional assumptions on the decision process, examples of which are provided.

The model is similar to Apps and Rees's, though, for the sake of consistency, I adopt the same notation as in my 1992 paper. The household consists of two members, 1 and 2. Respective demands for leisure are denoted by L_i, market labor supplies by l_i, and wages by w_i; let m_1 and m_2 be the members' respective nonlabor incomes.[1]

[1] What is actually needed for identification is in fact the existence of at least one "distribution factor" (see Bourguignon, Browning, and Chiappori 1994), i.e., factors that influence the decision process but neither the preferences nor the budget constraint. If m_1 and m_2 can be assumed exogenous, they will play this role: while their sum has an income effect, the distribution should matter only for the decision process. But other variables could be used, e.g., extra environmental parameters à la McElroy (1990).

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There are two consumption goods: a market good x, whose price is set to one, and a domestic good y, which can be produced within the household. Let $h(t_1, t_2)$ be the production function of the domestic good, where t_i is member i's household work.[2] Following a remark made some years ago by Pollak and Wachter (1975), I assume, as in Apps and Rees, that h exhibits constant returns to scale.

All goods are privately consumed. Also, following my initial model, I consider the case of cross-sectional data. Then wages and incomes vary across households, but other prices are assumed to be constant over the sample. Accordingly, let us assume that labor supplies l_i and t_i are observed as functions of w_1, w_2 and m_1, m_2. Finally, I forget about the tax system and assume that budget sets are linear.

The household consists of two members. In what follows, I assume that preferences are either "egoistic," as in Apps and Rees, in which case U^i has the form

$$U^i(x_i, y_i, L_i), \qquad (1)$$

or "caring," in which case each agent maximizes an index of the form

$$W^i[U^1(x_1, y_1, L_1), U^2(x_2, y_2, L_2)]. \qquad (2)$$

Finally, decisions are made according to some intrahousehold process. Following the basic idea of the collective approach, I simply assume that the decision process, whatever its true nature, always generates Pareto-efficient outcomes; this framework was used in my initial models (Chiappori 1988, 1992) as well as in Apps and Rees (1988) and various other contributions (e.g., Bourguignon and Chiappori [1992] and Browning et al. [1994]; see, however, Udry [1996]). In the present framework, any efficient decision process can be interpreted as follows: members agree on some efficient production plan and some intrahousehold distribution of resources; then each member freely chooses his or her own leisure–domestic production–consumption bundle subject to the specific budget constraint he or she faces. Formally, production is determined by

$$\max_{t_1, t_2} p \cdot h(t_1, t_2) - w_1 t_1 - w_2 t_2, \qquad (3)$$

and member i maximizes

$$\max_{x_i, y_i, L_i} U^i(x_i, y_i, L_i) \qquad (4)$$

$$\text{subject to} \quad x_i + p \cdot y_i + w_i \cdot L_i = s_i.$$

[2] Note that production is not assumed to depend only on total time $t_1 + t_2$; i.e., I allow for differences in marginal productivity of labor. One advantage of this generalization is that it can account for partial specialization (time input being nonzero for each member).

Here, s_i stands for member i's "potential income," with

$$s = s_1 + s_2 = (w_1 + w_2) \cdot T + m_1 + m_2 + p \cdot h(t_1, t_2) - w_1 t_1 - w_2 t_2. \tag{5}$$

In other words, s_i is a given function of (w_1, w_2, m_1, m_2) that "summarizes" the decision process; it can be seen as the natural generalization of the sharing rule, as introduced in my 1992 paper, to the case of household production.

As will be clear in what follows, a crucial issue is whether good y can be bought and sold on the market or is household specific. In the former case, let p denote the market price of the domestic good (p is then *exogenous* for the household) and z denote the quantity sold (if positive) or bought (if negative) on the market, so that the household's budget constraint is

$$x_1 + x_2 \leq w_1 l_1 + w_2 l_2 + m_1 + m_2 + p \cdot z.$$

In the second case, $z = 0$ for all households. As mentioned by Apps and Rees, we can still define a price p for the domestic good. The difference, however, is that p is now *endogenous* to the household preferences and decision process; technically, it has to be estimated as a function of the various parameters.

II. Is the Domestic Good Marketable?

Since the two cases above lead to totally opposite results, a quick discussion of their empirical relevance may be useful.

A. *The Case for a Marketable Domestic Good*

Farm Household

Whether the domestic good can be assumed marketable depends on the interpretation that is given to the notion of domestic production. In most cases, this assumption is in fact quite natural. The best example is agricultural households. In all developing countries (and in most developed ones as well), rural households have an agricultural production activity. Models of household production based on the maximization of a unique household utility, subject to a budget set and a production constraint, have become basic tools for the analysis of farm households.[3]

In the standard framework, as studied by many authors (including

[3] See Singh, Squire, and Strauss (1986) for a comprehensive survey of this literature.

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Lau, Lin, and Yotopoulos [1978], Barnum and Squire [1979], and others), markets are complete, and households are price takers for every commodity (including labor) they buy, sell, or consume. This means, in particular, that household production (consisting of commodities such as rice and coffee) can always be sold on sufficiently well-organized markets. As Singh et al. (1986, p. 3) put it, "most households in agricultural areas produce partly for sale and partly for their own consumption." A well-known consequence is the existence of a "separation" property between production and consumption. Production can be analyzed as stemming from the maximization of profit at market prices. In the same way, consumption is represented by utility maximization under a traditional budget constraint, again at market prices. The key point is that the household can make its production decision independently of its consumption and labor supply decisions.

As will be clear later on, the results of the next section rely on exactly the same separability property as in the standard "unitary" model of a farm household. The present paper can in fact be viewed as an extension of this literature to a collective framework, in which each household member is represented by specific preferences instead of being aggregated within the somewhat ad hoc fiction of a single household utility. The good news is that the nice properties of the standard, complete market framework (in terms of estimation in particular) do extend to the collective setting.

Domestic Production in Nonrural Households

While agricultural households do constitute a standard area of application of household production, it is by no means the only one. The general relevance of the concept of domestic production has been emphasized by many authors, starting with Becker (1991). In many cases, the same goods or services can also be bought on outside markets, at a given price. For example, meals can be taken at home or at a restaurant, children can be kept at home or in kindergartens, and so on. In that case, the corresponding price will play exactly the role described in the formal model: it will determine the trade-off between internal production and outside trade. Specifically, household members will devote their time to domestic production up to the point at which marginal productivity equals the ratio of the wage to the price of the good.[4]

[4] In some cases, the goods traded on outside markets are not perfect substitutes for domestic goods. Albeit this distinction may seem excessively subtle in the elementary framework of a standard labor supply model (where, for simplicity, all outside commodities are aggregated within a single good x, no fixed costs are considered, etc.), we may note that the argument above can be seen as an approximation: marginal

As another illustration, take the case of house cleaning. The trade-off faced by household members here is either doing the job themselves or hiring someone to do the work. Then a natural unit of measure for household productive output is the number of outside hours, and p can be interpreted as the market price of one hour of maid services. One could argue that a slight distinction should be introduced here, because hiring a maid should be interpreted as buying a productive input (rather than directly buying the output). But this fact can easily be reconciled with the framework at stake. Assume, for instance, that the output is actually produced from the work of the maid (who receives a wage s) plus some inputs (bought on the markets at a price q) according to some constant returns to scale technology. Then the production side defines the output price p as a *fixed* function of s and q; the crucial point is that p is again exogenous for the household.[5]

B. *The Case for a Nonmarketable Domestic Good*

While the complete market can be considered as the benchmark case, there may be various situations in which this assumption fails to hold true. Becker (1991, p. 24), for instance, considers such domestic goods as "prestige and esteem, health, altruism, envy, and pleasures of the senses," most of which (except perhaps the last one) can hardly be bought on outside markets. Some works on rural households have introduced the concept of Z-goods,[6] for which no market may exist (though this point has been disputed; see, e.g., Anderson and Leiserson [1980]). Note that, in both cases, not only is the output nonmarketable, but it is not observable in general (at least in a measurable way), which generates difficult identification problems.

Somewhat similar is the case in which, because of market imperfections of some kind, corner solutions obtain. Assume, for instance, that a given commodity is bought and sold at different prices, because of transaction costs, taxes, information asymmetries, or what-

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productivity has to be "close to" the wage/price ratio, the proximity level depending on the degree of substitutability between the commodities. It seems clear, however, that the dispersion between wages in a cross-sectional analysis will exceed, by and large, the gap between market and intrahousehold prices of domestic goods so that the latter distinction can probably be forgotten for empirical purposes.

[5] This assumes that the same technology is available to all households or, in a more realistic way, that differences in technologies can be neglected with regard to the huge dispersion between wages. If not—say, because differences in capital stocks across households are important enough—then an empirical estimation will have to control for the stock of durables in the usual way.

[6] See Hymer and Resnick (1969) and Singh et al. (1986) for a general discussion.

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ever. A fraction of households will then exhibit a zero net demand. The shadow price of the domestic good, in that case, will lie somewhere between the buying and the selling price, but the exact value will be endogenous to household behavior as a whole.

As is well known, a consequence of missing markets is that the separability property no longer holds; household decisions cannot be modeled as recursive, as in the previous case. Estimation of nonseparable models is known to be a much more difficult task.

III. The Complete Market Case

The first question now is whether, from the observation of t_i and l_i, we can recover s_i as a function of the relevant variables. In addition, we would like to derive testable restrictions on observed behavior. Specifically, we are looking for conditions that have to be fulfilled by arbitrary functions t_i and l_i (or L_i) of (w_1, w_2, m_1, m_2) to be a solution of (3) and (4) for well-chosen functions U^i, h, s_1, and s_2.

Both purposes appear to be feasible. Define, first,

$$Z_i = \frac{\partial L_i/\partial m_1}{\partial L_i/\partial m_2}. \tag{6}$$

Note that $Z_i = 1$ if income is pooled. In what follows, we assume that this is not the case, so that $Z_i \neq 1$. The main result can be stated as follows.

THEOREM 1. Let z, t_i, and L_i be given functions of (w_1, w_2, m_1, m_2), and assume that $Z_1 \neq 1$, $Z_2 \neq 1$, and $Z_1 \neq Z_2$. Then the following results hold true: (1) One can find a list of necessary and sufficient conditions for the existence of U^1, U^2, and h such that (L_1, L_2, t_1, t_2, z) are solutions of (3) and (4) for well-chosen functions U^i, h, s_1, and s_2 (they are given by conditions 1–7 in App. A). (2) For any functions satisfying these conditions, the sharing rule is identified up to an additive constant. In addition, individual consumption can be recovered up to an additive constant.

The complete proof is in Appendix A. I shall briefly sketch the basic steps. First, the observation of input demands $t_i(w_1, w_2)$ allows one to identify the production function h up to a multiplicative constant. Now, from (4), L_i can be written as some (Marshallian demand) function of w_i and s_i. From the observation of Z_i, one can identify the partials with respect to s_i. Finally, differentiating (5) in w_1 and w_2 leads to the identification of wage effects.

The properties of the complete market case are thus quite interesting. If one has a data base in which the time spent in household production by each member is recorded and distinct, exogenous income sources are observed, the conclusions of the initial model can readily be extended. Under the same assumptions (private consumptions and "caring" preferences), one can identify the sharing rule up to an additive constant, and individual, Marshallian demands for leisure can then be recovered. Furthermore, testable restrictions are generated on both market and household labor supply functions.

IV. The Incomplete Market Case

A. *Partial Identification of the Sharing Rule*

Let us now turn to the incomplete market case. In the maximization problems, p should no longer be interpreted as the actual price of the domestic good on some outside market, but rather as its shadow price. This is important because shadow prices are *endogenous* to household behavior. As such, p depends on wages and nonlabor income as well as on preferences and the decision process. How will this fact modify the results of the previous section?

I first consider the productive side. As before, the assumption of constant returns to scale allows the production function to be recovered up to a multiplicative constant. In addition, the output price is a given function of wages that can be estimated up to the same constant.[7] Note that p is homogeneous of degree one and depends only on wages.

I can now state the main result of this section.

THEOREM 2. Let t_i and L_i be given functions of (w_1, w_2, m_1, m_2). Assume that h exhibits constant returns to scale and that $Z_1 \neq 1$, $Z_2 \neq 1$, and $Z_1 \neq Z_2$. Then the following results hold true: (1) If (L_1, L_2, t_1, t_2, z) are solutions of (3) and (4) for well-chosen functions U^i, h, s_1, and s_2, then necessarily the ratio $\tau = t_1/t_2$ is a function of w_1/w_2 alone. (2) For any functions satisfying these conditions, the sharing rule is identified up to an additive function of (w_1, w_2). That is, if $s_i(w_1, w_2, m_1, m_2)$ is a sharing rule compatible with observed behavior, then any alternative solution must have the form

$$s_i'(w_1, w_2, m_1, m_2) = s_i(w_1, w_2, m_1, m_2) + f(w_1, w_2) \qquad (7)$$

for some arbitrary function f.

[7] To see why, note that if $h(t_1, t_2) = t_1 H(\tau)$, where $\tau = t_2/t_1$, then H is identified up to a multiplicative constant, which is obviously not identifiable. Then p is given by

$$p = \frac{w_i}{\partial h/\partial t_i} = \frac{w_2}{H'[\tau(w_1/w_2)]} = p(w_1, w_2).$$

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The proof is given in Appendix B.

The previous result suggests two remarks. First, endogeneity of the domestic price has a cost in terms of identification; indeed, the sharing rule can be estimated only up to an additive function of wages. This finding generalizes proposition 3 in Apps and Rees. Second, while complete identification is out of reach, there is still much to be learned from the observation of labor supplies. For one thing, the effect of nonlabor incomes on the sharing rule is exactly identified. The reason is that a change in m_1 and m_2 that leaves wages constant will not modify the price of the domestic good; the second-round effects alluded to above are thus ruled out. Note that this generalizes proposition 4 in Apps and Rees since we do not need to assume that the production function is linear. This is important information, especially in view of policies based on cash subsidies whose effect is exactly to alter the members' respective nonlabor incomes. For instance, we may be able to assess the consequences of a lump-sum distribution between husband and wife on the intrahousehold distribution of resources, even though the distribution is only partially identified. This has important implications, in particular for the "targeted" programs discussed in Haddad and Kanbur (1992); for an empirical investigation along these lines, see Lundberg, Pollak, and Wales (1995).

B. *Parametric versus Nonparametric Identification*

An important consequence of this result is that one has to be extremely cautious when identifying a collective model with household production. In particular, a parametric approach can be dangerously misleading. Assume that a given, parametric structural model generates a reduced form such that all the parameters of the model are identified. It may still be the case that other, *functionally different* structural models lead to the *same* reduced form. Then there is no reason to single out the initial model; as a consequence, all welfare recommendations based on the parametric identification are invalid.

This can be readily seen in the following example. Assume that utilities have the linear expenditure system form:

$$U^i(x_i, y_i, L_i) = \alpha_i \log(x_i - \bar{x}_i) + \beta_i \log(y_i - \bar{y}_i) + \gamma_i \log(L_i - \bar{L}_i),$$

with $\alpha_i + \beta_i + \gamma_i = 1$; the corresponding Marshallian demands are $\Lambda_i(p, w_i, s_i)$, with

$$w_i\Lambda_i = w_i\bar{L}_i + \gamma_i(s_i - \bar{x}_i - p\bar{y}_i - w_i\bar{L}_i).$$

Assume that the production function is Cobb-Douglas:

$$h(t_1, t_2) = (t_1)^{\delta}(t_2)^{1-\delta},$$

and take for the sharing rule the following form:

$$s_1(w_1, w_2, m_1, m_2) = m_1 + w_1 T + S_o . w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2},$$

$$s_2(w_1, w_2, m_1, m_2) = m_2 + w_2 T - S_o . w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2}.$$

Then one can compute individual demand and labor supply functions. On the production side,

$$\frac{t_2}{t_1} = \frac{1-\delta}{\delta}\frac{w_1}{w_2}$$

identifies δ; then

$$p = K(w_1)^{\delta}(w_2)^{1-\delta},$$

where K is a known function of δ. Consider, now, demands for leisure:

$$w_1 L_1 = -\gamma_1 \bar{x}_1 + [(1-\gamma_1)\bar{L}_1 + \gamma_1 T] w_1 - \gamma_1 \bar{y}_1 K(w_1)^{\delta}(w_2)^{1-\delta} + \gamma_1 (m_1 + S_o . w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2}),$$

$$w_2 L_2 = -\gamma_2 \bar{x}_2 + [(1-\gamma_2)\bar{L}_2 + \gamma_2 T] w_2 - \gamma_2 \bar{y}_2 K(w_2)^{\delta}(w_2)^{1-\delta} + \gamma_2 (m_2 - S_o . w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2}).$$

The main remark is that *all parameters are identifiable from the demand for leisure equation* (as can be readily checked). A naive econometrician might conclude that he has recovered the "true" utility functions and the "true" sharing rule. Even worse, he may try to deduce normative recommendations about welfare policies. Now, all this is a pure illusion. For assume that the true sharing rule is in fact

$$\bar{s}_1(w_1, w_2, m_1, m_2) = m_1 + w_1 T + S_o . w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2} + \nu_1 w_1 - \nu_2 w_2,$$

$$\bar{s}_2(w_1, w_2, m_1, m_2) = m_2 + w_2 T - S_o\, w_1^{\alpha_1} w_2^{\alpha_2} m_1^{\beta_1} m_2^{\beta_2} - \nu_1 w_1 + \nu_2 w_2,$$

where the ν_i are arbitrary, nonnegative scalars. Define the functions λ_i, $i = 1, 2$, by

$$\lambda_i(w_i, p, s_i) = \Lambda_i[w_i, p, s_i - \nu_i w_i + \nu_j K^{-1/(1-\delta)} p^{1/(1-\delta)} w_i^{-\delta/(1-\delta)}].$$

From standard consumer theory, we know that there exists some utility function for which λ_i is the Marshallian demand for leisure[8]

[8] The only condition is negativity of the diagonal Slutsky term, which is fulfilled provided that $\partial\Lambda_i/\partial s > 0$.

(of course, it will not have the linear expenditure system form). And it can be readily checked that the $\lambda\ \ _i$, together with the $\bar{s}_i$, generate exactly the same reduced forms as above. Of course, the interpretation of the coefficients is totally different, and no normative conclusion will be robust to the new specification, though it is not empirically distinguishable from the initial one.

C. *Particular Assumptions on the Decision Process*

Finally, it should be noted that the model considered above is extremely general: it only assumes efficiency of household decisions and imposes no restriction on the location of the final outcome on the Pareto frontier. A natural conjecture is that stronger results (and possibly full identification) would obtain, should some additional structure be introduced. A possible solution is to impose some structural restriction on the decision process. As an illustration, consider the following assumption.

ASSUMPTION H. The sharing rules depend only on total income brought by each member; that is, s_i must be a function of $w_1 T + m_1$ and $w_2 T + m_2$ alone.

Then, again, the sharing rule can be fully identified. To see why, note that, from (7), we know that, for any solution s'_i,

$$\frac{\partial s'_i}{\partial w_i} = \frac{\partial s_i}{\partial w_i} + \frac{\partial f}{\partial w_i} \tag{8}$$

and

$$\frac{\partial s'_i}{\partial m_i} = \frac{\partial s_i}{\partial m_i}. \tag{9}$$

Also, assumption H requires that

$$\frac{\partial s'_i}{\partial w_i} = T\frac{\partial s'_i}{\partial m_i}.$$

It follows that

$$\frac{\partial s_i}{\partial w_i} + \frac{\partial f}{\partial w_i} = T\frac{\partial s_i}{\partial m_i}.$$

Hence,

$$\frac{\partial f}{\partial w_i} = T\frac{\partial s_i}{\partial m_i} - \frac{\partial s_i}{\partial w_i}, \tag{10}$$

and f is identified up to an additive constant. In words, among all solutions of the form (7), one at most is compatible with assumption

H. In addition, assumption H generates new restrictions (second cross derivatives, as computed from [10], must be equal). So the identifying assumption is independently testable, as requested by standard methodological considerations.

Of course, assumption H is only an example. Basically, any restriction stating that s_i depends on (w_1, w_2, m_1, m_2) only through given functions $A_1(w_1, w_2, m_1, m_2)$ and $A_2(w_1, w_2, m_1, m_2)$ will lead to similar conclusions.[9] Finally, a more general question is the following: Can we think of particular, axiomatic restrictions on the decision process itself that will lead to complete identification of the sharing rule even in the nonmarketable case? Assume, for instance, that the process can adequately be described with the help of some specific cooperative bargaining concept (Nash bargaining, Kalai-Smorodinsky, Rubinstein's "shrinking cake," or whatever). Would this help identification in some way? This is an open question to which further work will be devoted. In addition, empirical applications are obviously needed, following the initial attempts by Blundell et al. (1996) and Fortin and Lacroix (1996).

Appendix A

Proof of Theorem 1

A. The Production Side

I start by considering production decisions. As stated above, the household maximizes profit:

$$\max_{t_1, t_2} p \cdot h(t_1, t_2) - w_1 t_1 - w_2 t_2, \tag{A1}$$

where $h(t_1, t_2) = t_1 H(\tau)$, with $\tau = t_2/t_1$; note that τ, like t_1 and t_2, can be observed as a function of (w_1, w_2, m_1, m_2). Now efficiency requires that

$$\frac{\partial h/\partial t_1}{\partial h/\partial t_2} = \frac{H(\tau)}{H'(\tau)} - \tau = \frac{w_1}{w_2}.$$

[9] To see why, simply note that if this is the case, then the gradient of the sharing rule must belong to the subspace spanned by the gradients of A_1 and A_2. Now, among the solutions of the form (7), again at most one is compatible with this property. Indeed, assume that $Ds' = \alpha \cdot DA_1 + \beta \cdot DA_2$, where Dg denotes the gradient of g. Derivatives with respect to m_1 and m_2 give

$$\frac{\partial s'}{\partial m_i} = \alpha \cdot \frac{\partial A_1}{\partial m_i} + \beta \cdot \frac{\partial A_2}{\partial m_i}.$$

If the A_i are regular, in the sense that

$$\frac{\partial A_1/\partial m_1}{\partial A_1/\partial m_2} \neq \frac{\partial A_2/\partial m_1}{\partial A_2/\partial m_2},$$

these equations identify α and β. Then derivatives with respect to w_1 and w_2 identify the partials of f and, hence, the function itself up to an additive constant.

This differential equation defines H up to a multiplicative constant, which is obviously not identifiable unless total output is observed. Here, τ is defined as a function of w_1/w_2 alone, a fact that provides a first testable restriction.

CONDITION 1. t_2/t_1 must be a function of w_1/w_2 alone.

B. Demand for Leisure: Income Effects

We now come to labor supply, as determined by the program

$$\max_{x_i, y_i, L_i} U^i(x_i, y_i, L_i) \tag{A2}$$

$$\text{subject to} \quad x_i + p \cdot y_i + w_i \cdot L_i = s_i.$$

A first remark is that demand for leisure must have the form

$$L_i(w_1, w_2, m_1, m_2) = \lambda_i(w_i, s_i), \tag{A3}$$

where λ_i is the Marshallian demand derived from U^i. It follows that

$$\frac{\partial L_i}{\partial m_j} = \frac{\partial \lambda_i}{\partial s_i}\frac{\partial s_i}{\partial m_j}; \tag{A4}$$

hence,

$$Z_i = \frac{\partial L_i/\partial m_1}{\partial L_i/\partial m_2} = \frac{\partial s_i/\partial m_1}{\partial s_i/\partial m_2}, \tag{A5}$$

where Z_i is observable. Also, we know that

$$s = s_1 + s_2 = (w_1 + w_2) \cdot T + m_1 + m_2;$$

hence,

$$\frac{\partial s_1}{\partial m_j} + \frac{\partial s_2}{\partial m_j} = 1, \tag{A6}$$

for $j = 1, 2$. From (A4) and (A6), the partials of s with respect to m can be exactly identified.

LEMMA 1. Assume that $Z_1 \neq Z_2$. Then

$$\frac{\partial s_1}{\partial m_1} = Z_1\frac{Z_1 - 1}{Z_2 - Z_1},$$

$$\frac{\partial s_1}{\partial m_2} = \frac{Z_2 - 1}{Z_2 - Z_1},$$

$$\frac{\partial s_2}{\partial m_1} = Z_2\frac{Z_1 - 1}{Z_1 - Z_2},$$

$$\frac{\partial s_2}{\partial m_2} = \frac{Z_1 - 1}{Z_1 - Z_2}.$$

Note that if income is pooled, $Z_1 = Z_2 = 1$ and the partials are indeterminate.

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Of course, the derivation of derivatives immediately generates cross-derivative restrictions.

CONDITION 2. The partials given by lemma 1 must satisfy

$$\frac{\partial^2 s_i}{\partial m_1 \partial m_2} = \frac{\partial^2 s_i}{\partial m_2 \partial m_1}.$$

In addition, (A4) gives the partials $\partial\lambda/\partial m$:

$$\frac{\partial \lambda_i}{\partial s_i} = \frac{\partial L_i/\partial m_j}{\partial s_i/\partial m_j},$$

which does not depend on j by construction. This function must depend only on (w_i, s_i); in particular, the following condition must hold.

CONDITION 3. The functions L and s must satisfy

$$\frac{\dfrac{\partial}{\partial m_1}\left(\dfrac{\partial L_i/\partial m_j}{\partial s_i/\partial m_j}\right)}{\dfrac{\partial}{\partial m_2}\left(\dfrac{\partial L_i/\partial m_j}{\partial s_i/\partial m_j}\right)} = \frac{\partial s_i/\partial m_1}{\partial s_i/\partial m_2}.$$

C. Demand for Leisure: Wage Effects

From (A3), it follows that

$$\frac{\partial L_i}{\partial w_j} = \frac{\partial \lambda_i}{\partial s_i} \times \frac{\partial s_i}{\partial w_j}$$

for $i \neq j$. Hence,

$$\frac{\partial s_1}{\partial w_2} = \frac{\partial L_1/\partial w_2}{\partial \lambda_1/\partial s_1} = \frac{\partial s_1}{\partial m_1} \times \frac{\partial L_1/\partial w_2}{\partial L_1/\partial m_1}, \tag{A7}$$

where $\partial s_1/\partial m_1$ is given by lemma 1. Similarly,

$$\frac{\partial s_2}{\partial w_1} = \frac{\partial s_2}{\partial m_1} \times \frac{\partial L_2/\partial w_1}{\partial L_2/\partial m_1}. \tag{A8}$$

Finally, the own-wage effect can be recovered from (A5):

$$\frac{\partial s_1}{\partial w_1} = \frac{\partial s}{\partial w_1} - \frac{\partial s_2}{\partial w_1} = T - \frac{\partial s_2}{\partial w_1} \tag{A9}$$

and

$$\frac{\partial s_2}{\partial w_2} = T - \frac{\partial s_1}{\partial w_2}. \tag{A10}$$

It follows that all partials of s are identified: s can be recovered up to an additive constant. Of course, new testable restrictions arise.

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CONDITION 4. Cross-derivative restrictions on s are

$$\frac{\partial^2 s_i}{\partial w_1 \partial w_2} = \frac{\partial^2 s_i}{\partial w_2 \partial w_1},$$

$$\frac{\partial^2 s_i}{\partial m_k \partial w_j} = \frac{\partial^2 s_i}{\partial w_j \partial m_k}.$$

Also, $\partial \lambda_i / \partial s_i$ depends on w_j only through s_i, which implies the following condition.

CONDITION 5.

$$\frac{\dfrac{\partial}{\partial w_j}\left(\dfrac{\partial L_i/\partial m_1}{\partial s_i/\partial m_1}\right)}{\dfrac{\partial}{\partial m_k}\left(\dfrac{\partial L_i/\partial m_1}{\partial s_i/\partial m_1}\right)} = \frac{\partial s_i/\partial w_j}{\partial s_i/\partial m_k}.$$

Finally, we can now recover $\partial \lambda_i / \partial w_i$; indeed, note that

$$\frac{\partial L_i}{\partial w_i} = \frac{\partial \lambda_i}{\partial w_i} + \frac{\partial \lambda_i}{\partial s_i} \times \frac{\partial s_i}{\partial w_i},$$

which implies that

$$\frac{\partial \lambda_i}{\partial w_i} = \frac{\partial L_i}{\partial w_i} - \frac{\partial s_i}{\partial w_i} \times \frac{\partial L_i/\partial m_1}{\partial s_i/\partial m_1}. \tag{A11}$$

This partial must be a function of w_i and s_i alone, which generates the usual conditions; furthermore, it must be compatible with the partials $\partial \lambda / \partial s$ previously computed. Hence, thc following condition must hold.

CONDITION 6.

$$\frac{\dfrac{\partial}{\partial w_j}\left(\dfrac{\partial \lambda_i}{\partial w_i}\right)}{\dfrac{\partial}{\partial m_k}\left(\dfrac{\partial \lambda_i}{\partial w_i}\right)} = \frac{\partial s_i/\partial w_j}{\partial s_i/\partial m_k},$$

$$\frac{\dfrac{\partial}{\partial m_1}\left(\dfrac{\partial \lambda_i}{\partial w_i}\right)}{\dfrac{\partial}{\partial m_2}\left(\dfrac{\partial \lambda_i}{\partial w_i}\right)} = \frac{\partial s_i/\partial m_1}{\partial s_i/\partial m_2},$$

$$\frac{\partial^2 \lambda_i}{\partial w_i \partial s_i} = \frac{\partial^2 \lambda_i}{\partial s_i \partial w_i}.$$

However, Slutsky conditions do not impose new restrictions on λ because we do not observe changes in p.

D. Recovering Private Consumptions

Once the sharing rule has been identified, it is easy to recover the individual consumptions of both market and domestic goods. To see how, note that

$$y(w_1, w_2, m_1, m_2) = Y_1(w_1, s_1) + Y_2(w_2, s_2),$$

where y is observed but the Marshallian demands Y_1 and Y_2 are not. Now, compute the derivatives with respect to m_1 and m_2:

$$\frac{\partial y}{\partial m_i} = \frac{\partial Y_1}{\partial s_1} \times \frac{\partial s_1}{\partial m_i} + \frac{\partial Y_2}{\partial s_2} \times \frac{\partial s_2}{\partial m_i}. \tag{A12}$$

Then we have the following lemma.

LEMMA 2. Assume $Z_1 \neq 1$, $Z_2 \neq 1$, and $Z_1 \neq Z_2$. Then $\partial Y_1/\partial s_1$ and $\partial Y_2/\partial s_2$ can be identified from (A12).

Proof. Equation (A12) provides a linear system with the two partials as unknowns. The determinant of the system is equal to $(Z_1 - 1) \cdot (Z_2 - 1) \cdot (Z_1 - Z_2)$ and hence is nonzero under the assumption.

Finally, once the partials with respect to s have been recovered, the two remaining partials, $\partial Y_1/\partial w_1$ and $\partial Y_2/\partial w_2$, follow immediately from

$$\frac{\partial y}{\partial w_i} = \frac{\partial Y_i}{\partial w_i} + \frac{\partial Y_1}{\partial s_1} \times \frac{\partial s_1}{\partial w_i} + \frac{\partial Y_2}{\partial s_2} \times \frac{\partial s_2}{\partial w_i}. \tag{A13}$$

Hence the Marshallian demands are known up to an additive constant.

As before, this implies two kinds of restrictions; namely, those derivatives must be functions of (w_i, s_i) alone and satisfy cross-derivative restrictions. This leads to the final bunch of testable restrictions.

CONDITION 7.

$$\frac{\frac{\partial}{\partial m_1}\left(\frac{\partial Y_i}{\partial w_i}\right)}{\frac{\partial}{\partial m_2}\left(\frac{\partial Y_i}{\partial w_i}\right)} = \frac{\partial s_i/\partial w_j}{\partial s_i/\partial m_k},$$

$$\frac{\frac{\partial}{\partial w_j}\left(\frac{\partial Y_i}{\partial w_i}\right)}{\frac{\partial}{\partial m_k}\left(\frac{\partial Y_i}{\partial w_i}\right)} = \frac{\partial s_i/\partial w_j}{\partial s_i/\partial m_k} \quad \text{for } j \neq i,$$

$$\frac{\frac{\partial}{\partial m_1}\left(\frac{\partial Y_i}{\partial s_i}\right)}{\frac{\partial}{\partial m_2}\left(\frac{\partial Y_i}{\partial s_i}\right)} = \frac{\partial s_i/\partial m_1}{\partial s_i/\partial m_2},$$

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$$\frac{\dfrac{\partial}{\partial w_j}\left(\dfrac{\partial Y_i}{\partial s_i}\right)}{\dfrac{\partial}{\partial m_k}\left(\dfrac{\partial Y_i}{\partial s_i}\right)} = \frac{\partial s_i/\partial w_j}{\partial s_i/\partial m_k} \quad \text{for } j \neq i,$$

$$\frac{\partial_i^2 Y}{\partial w_i \partial s_i} = \frac{\partial_i^2 Y}{\partial s_i \partial w_i}.$$

Finally, the x_i can be recovered either through the same technique or, more directly, from the budget constraints, since the s_i are known. Note, in particular, that no additional restrictions appear at this stage: the x_i will automatically fulfill the conditions of the previous type from the budget constraint. This terminates the proof.

Appendix B

Proof of Theorem 2

Let us consider consumption behavior. The program becomes

$$\max_{x_i, y_i, L_i} U^i(x_i, y_i, L_i) \tag{B1}$$

$$\text{subject to } x_i + p(w_1, w_2) \cdot y_i + w_i \cdot L_i = s_i.$$

As before, demand for leisure must have the form

$$L_i(w_1, w_2, m_1, m_2) = \lambda_i(w_i, p(w_1, w_2), s_i), \tag{B2}$$

where λ_i is the Marshallian demand derived from U^i and

$$s = s_1 + s_2 = (w_1 + w_2) \cdot T + m_1 + m_2. \tag{B3}$$

Assuming that $\partial L_i/\partial m_j \neq 0$ for some j, we can locally invert (B2) as

$$s_i(w_1, w_2, m_1, m_2) = F_i[w_i, p(w_1, w_2), L_i(w_1, w_2, m_1, m_2)]$$

for some function F_i that has to be estimated.

Equation (B3) becomes

$$\begin{aligned} &F_1[w_1, p(w_1, w_2), L_1(w_1, w_2, m_1, m_2)] \\ &+ F_2[w_2, p(w_1, w_2), L_2(w_1, w_2, m_1, m_2)] = (w_1 + w_2) \cdot T + m_1 + m_2. \end{aligned} \tag{B4}$$

Now, let (F_1, F_2) and (F_1', F_2') be two solutions of this equation, and define $G_i = F_i - F_i'$. I show that G_i can be written as a function of wages alone. Since (B4) is linear, we know that the G_i must satisfy the homogeneous equation

$$G_1[w_1, p(w_1, w_2), L_1(w_1, w_2, m_1, m_2)]$$

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$$+ G_2[w_2, p(w_1, w_2), L_2(w_1, w_2, m_1, m_2)] = 0.$$

Differentiating in m_i gives

$$\frac{\partial G_1}{\partial L_1}\frac{\partial L_1}{\partial m_i} + \frac{\partial G_2}{\partial L_2}\frac{\partial L_2}{\partial m_i} = 0.$$

Since this system has full rank, its only solution is

$$\frac{\partial G_1}{\partial L_1} = \frac{\partial G_2}{\partial L_2} = 0;$$

hence $G_i = G_i[w_i, p(w_1, w_2)] = f_i(w_1, w_2)$.

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ANNEXE 2

Modèle collectif et participation :

Note : la recherche est en cours. Les notes ci-jointes indiquent d'une part le résultat du modèle théorique (y compris le théorème central), d'autre part l'avancement de la partie économétrique. Sur ce dernier point, la note interne jointe indique le résultat d'identification de la forme linéarisée.

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1. Introduction

2. The theory

2.1. Preferences and decision process

The framework is identical to that in Chiappori (1989,1992). We consider a simple labor supply model within a two-member household; let h^i and c^i respectively denote member i's labor supply (with $0 \leq h^i \leq 1$) and consumption of a private Hicksian commodity c (with $c^1 + c^2 = C$), the price of which is set to one; and let w_1, w_2 and y denote respective wages and the household's non labor income. Preferences are taken of the 'egoistic' type; i.e., member i's utility can be written $U^i(1 - h^i, c^i)$, where U^i is continuously differentiable, strictly monotone and strongly quasi-concave; as in Chiappori (1992), this can be extended at no cost to 'caring' preferences of the form $W^i\left[U^1, U^2\right]$. Finally, throughout the paper, we do not consider domestic production[1]. Also, taxation is disregarded, and budget frontiers are assumed linear.

Following the collective approach, we postulate that the household only takes Pareto-efficient decisions. That is, for any (w_1, w_2, y), there exist some $\bar{u}^2(w_1, w_2, y)$ such that the (h^i, c^i) are solution of the program :

$$\max_{L^1, L^2, c^1, c^2} U^1\left[1 - h^1, c^1\right] \tag{2.1}$$

$$U^2\left[1 - h^2, c^2\right] \geq \bar{u}^2(w_1, w_2, y)$$

$$c^1 + c^2 = w_1.h^1 + w_2.h^2 + y$$

$$0 \leq h^i \leq 1, \quad i = 1, 2$$

In words : for any given point (w_1, w_2, y), (2.1) describes the set of Pareto-efficient outcomes; this set, as usual, is parametrized by some $\bar{u}^2$. Any particular decision rule is reflected by a particular value of $\bar{u}^2$; in general, the latter will depend on (w_1, w_2, y).

In what follows, $\bar{u}^2$ is assumed to satisfy the following, regularity assumption:

[1]See Chiappori (1997) for a detailed investigation of domestic production in the standard, 'collective' model of household labor supply

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Assumption R : $\bar{u}^2$ is a *function* of (w_1, w_2, y). Moreover, it is twice continuously differentiable.

This assumption means that (i) the decision process always leads to some unique, well-defined outcome, and (ii) the latter is smooth, in the sense that infinitesimal changes in wages or non labor incomes cannot result in drops in utility for one of the members.

A consequence is that the solutions of (2.1) are well-behaved demand and labor supply functions (rather than correspondances).

2.2. The sharing rule

We now come to the decentralization of Pareto optima. The general result is the following :

Proposition 2.1. *Take any twice differentiable function $\bar{u}^2(w_1, w_2, y)$, and let $\bar{c}^1(w_1, w_2, y), \bar{c}^2(w_1, w_2, y), \bar{h}^1(w_1, w_2, y), \bar{h}^2(w_1, w_2, y)$ be the solutions of (2.1). Then there exists a sharing rule, i.e. two mappings $\rho^1(w_1, w_2, y)$ and $\rho^2(w_1, w_2, y)$, such that :*

- $\rho^1(w_1, w_2, y) + \rho^2(w_1, w_2, y) = y$
- $\bar{c}^i(w_1, w_2, y)$ *and* $\bar{h}^i(w_1, w_2, y)$ *are solutions of :*

$$\max_{L^i, c^i} U^i \left[1 - h^i, c^i\right] \tag{2.2}$$

$$c^i = w_i . h^i + \rho^i(w_1, w_2, y)$$
$$0 \leq h^i \leq 1$$

Any decision process leading to Pareto-efficient outcomes can thus be equivalently interpreted as a two-stage process. At stage one, agents share non-labor income between them (of course, this choice may, and in general will, depend on the current values of wages and non labor income). At stage two, each member independently maximizes his (her) welfare, subject to the budget constraint stemming from stage one. In particular, member i's labor supply is of the form :

$$h^i(w_1, w_2, y) = H^i \left[w_i, \rho^i(w_1, w_2, y)\right]$$

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where H^i is the Marshallian labor supply function associated to U^i.

The proof of Proposition 1 is straightforward : it is a particular case of standard results on the decentralizability of competitive equilibria[2]. Note, however, that in the previous statement, no assumption is made on the nature of the choice set for labor supplies; in particular, the result also applies to the case of discrete (participation) choices. However, the ρ^i may not be differentiable, or even continuous. We shall see below, in particular, that whenever the labor supply decision is dichotomous (i.e., $h^i = 0$ or 1), then the continuity of $\bar{u}^2$ implies that ρ^i is discontinuous on the participation frontier.

In what follows, the 'structural' model is defined by the functions U^1, U^2 and $\bar{u}^2$ (or, equivalently, U^1, U^2, ρ^1 and ρ^2). As in Chiappori (1992), two questions arise : what does this structure imply on the form of labor supply functions ? and to what extend is it possible to recover the structural model from the observation of labor supply ?

3. The case of corner solutions

3.1. Participation

We first consider the case where labor supplies can be observed as continuous functions of wages and non labor income. That is, we start from some given functions $h^1(w_1, w_2, y)$ and $h^2(w_1, w_2, y)$. These functions are in fact differentiable, except may be when they vanish. That is, there exist, for each member i, some frontier beyond which the solution of (3.2) is $h^i = 0$. In what follows, we assume, for simplicity, that this frontier can be interpreted in terms of reservation wages; technically :

Assumption W : for $i = 1, 2$, there exist a function $\gamma^i(w_j, y)$ (with $j \neq i$) such that :

$$h^i > 0 \Leftrightarrow \omega_i > \gamma^i(w_j, y)$$

Note that the functions γ^i can be empirically observed, and will be considered as given in the remaining of this section. It follows that the space of the (w_1, w_2, y) can be partitioned into four areas :

[2] As remarked for instance by Apps and Rees (1997). For a precise proof, see Chiappori (1992).

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- H_{11}, where $h^1 > 0$ and $h^2 > 0$; this is equivalent to

$$\omega_1 > \gamma^1(w_2, y) \text{ and } \omega_2 > \gamma^2(w_1, y)$$

- H_{10}, where $h^1 > 0$ and $h^2 = 0$; this is equivalent to

$$\omega_1 > \gamma^1(w_2, y) \text{ and } \omega_2 \leq \gamma^2(w_1, y)$$

- H_{01}, where $h^1 = 0$ and $h^2 > 0$; this is equivalent to

$$\omega_1 \leq \gamma^1(w_2, y) \text{ and } \omega_2 > \gamma^2(w_1, y)$$

- H_{00}, where $h^1 = 0$ and $h^2 = 0$; this is equivalent to

$$\omega_1 \leq \gamma^1(w_2, y) \text{ and } \omega_2 \leq \gamma^2(w_1, y)$$

(see Figure 1).

3.1.1. The extrapolation problem

The identification of the structural model from observed behavior will raise specific problems in some areas. Typically, in H_{00} no behavior can be observed at all, so nothing can be identified. However, as we shall see, both the individual utilitity functions and the sharing rule (or, equivalently, the function $\bar{u}^2$) can be recovered, up to an additive constant, in each other area. The standard solution, then, is to extrapolate to H_{00} the functions computed on H_{11}, H_{10} and H_{01}. For instance, assume that we adopt a parametric approach, where the functional form of, say, U^1, U^2 and $\bar{u}^2$ are chosen a priori. The corresponding parameters (but one) can be identified from observations in H_{11}; then, in principle, the structural model is known everywhere, although of course one has to assume that the functional form does not change over the space.

A nice feature of the 'extrapolation' technique, however, is that it is empirically testable. Indeed, consider, say, H_{10}. As we shall see, the knowledge of both member 1's labor supply h^1, *on* H_{10}, and the frontier γ^2 allows to identify the structural model (up to an additive constant), ***independently of any estimation made in any other area***. Now, one can readily compare these estimates to those obtained in H_{11}. If the 'extrapolation principle' is correct, then the latter should coincide. In a parametric setting, this provides an obvious specification test. Even within a non-parametric context, the assumption that $\bar{u}^2$ is continuously differentiable can be tested over the participation frontier, since we can independently compute its partials on both sides.

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3.2. Identification of the structural model

We now describe the identification procedure on each of the four areas defined above. In H_{11}, this is straightforward, since the results of Chiappori (1992) apply. It follows that preferences and the sharing rule can locally be recovered, up to an additive constant.

We now consider the case of H_{10} (that of H_{01} is exactly similar). First, since member 2 does participate, his utility is $U^2(c^2, 0)$, and we have that :

$$U^2\left(c^2, 1\right) = \bar{u}^2(w_1, w_2, y)$$

which can be inverted in :

$$c^2 = V^2\left[\bar{u}^2(w_1, w_2, y)\right] = \rho^2(w_1, w_2, y)$$

where V^2 is the inverse of the mapping $U^2(., 0)$. Now, from the results above, member 1's labor supply is of the form :

$$h^1(w_1, w_2, y) = H^1\left[w_1, y - \rho^2(w_1, w_2, y)\right]$$

where H^1 is the Marshallian labor supply function associated to U^1.

A first consequence is that, for any $(w_1, w_2, y) \in P$:

$$\frac{-\rho^2_{w_2}}{1-\rho^2_y} = \frac{h^1_{w_2}}{h^1_y} \doteq A(w_1, w_2, y) \tag{3.1}$$

where f_z stands for $\frac{\partial f}{\partial z}$. Note that the function h^1, hence the ratio A, are empirically observable. Hence (3.1) provides a first characterization of ρ^2.

Similarly, 2's labor supply takes the form :

$$h^2(w_1, w_2, y) = H^2\left[w_2, \rho^2(w_1, w_2, y)\right]$$

Since it vanishes on the frontier, we have that :

$$H^2\left[\gamma^2(w_1, y), \rho^2\left(w_1, \gamma^2(w_1, y), y\right)\right] = 0$$

We then use the following Lemma :

Lemma 3.1. *Take any point (w_1, w_2, y) on the participation frontier (i.e., such that $w_2 = \gamma^2(w_1, y)$), and assume that $\gamma^2_y \neq 0$. Then $\rho^2_y \neq 0$, and*

$$\frac{\gamma^2_{w_1}}{\gamma^2_y} = \frac{\rho^2_{w_1}}{\rho^2_y}$$

where all partials are taken at $(w_1, \gamma^2(w_1, y), y)$

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The nature of the assumptions we make is probably most relevant to households with no children, since the latter can be viewed as a public good. We should also note that our analysis should be viewed as conditional on a number of durable public goods such as housing. We view these expenditures as "sunk". All decisions made by individuals should be viewed conditional on these choices, which probably have a lot to do with the household formation decisions.

We use our methods to revisit family labour supply and test the basic implications of the theory.

2. The theory

2.1. Preferences and decision process

The framework is identical to that in Chiappori (1989,1992). We consider a simple labor supply model within a two-member household; let h^i and c^i respectively denote member i's labor supply (with $0 \leq h^i \leq 1$) and consumption of a private Hicksian commodity c (with $c^1 + c^2 = C$), the price of which is set to one; and let w_1, w_2 and y denote respective wages and the household's non labor income. Preferences are taken of the 'egoistic' type; i.e., member i's utility can be written $U^i(1-h^i, c^i)$, where U^i is continuously differentiable, strictly monotone and strongly quasi-concave; as in Chiappori (1992), this can be extended at no cost to 'caring' preferences of the form $W^i[U^1, U^2]$. Finally, throughout the paper, we do not consider domestic production[1]. Also, taxation is disregarded, and budget frontiers are assumed linear.

Following the collective approach, we postulate that the household only takes Pareto-efficient decisions. That is, for any (w_1, w_2, y), there exist some $\bar{u}^2(w_1, w_2, y)$ such that the (h^i, c^i) are solution of the program :

$$\max_{L^1,L^2,c^1,c^2} U^1\left[1-h^1, c^1\right] \tag{2.1}$$

$$U^2\left[1-h^2, c^2\right] \geq \bar{u}^2(w_1, w_2, y)$$

$$c^1 + c^2 = w_1.h^1 + w_2.h^2 + y$$

$$0 \leq h^i \leq 1, \quad i = 1, 2$$

[1]See Chiappori (1997) for a detailed investigation of domestic production in the standard, 'collective' model of household labor supply

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In words : for any given point (w_1, w_2, y), (2.1) describes the set of Pareto-efficient outcomes; this set, as usual, is parametrized by some $\bar{u}^2$. Any particular decision rule is reflected by a particular value of $\bar{u}^2$; in general, the latter will depend on (w_1, w_2, y).

In what follows, $\bar{u}^2$ is assumed to satisfy the following, regularity assumption:

Assumption R : $\bar{u}^2$ is a *function* of (w_1, w_2, y). Moreover, it is twice continuously differentiable.

This assumption means that (i) the decision process always leads to some unique, well-defined outcome, and (ii) the latter is smooth, in the sense that infinitesimal changes in wages or non labor incomes cannot result in drops in utility for one of the members. A consequence is that the solutions of (2.1) are well-behaved demand and labor supply functions (rather than correspondances).

Finally, throughout this paper, we assume that member 1 can freely choose her working hours, while 2 can only decide to participate (then $h^2 = 1$) or not ($h^2 = 0$).

In what follows, the 'structural' model is defined by the functions U^1, U^2 and $\bar{u}^2$ (or, equivalently, U^1, U^2, ρ^1 and ρ^2). As in Chiappori (1992), two questions arise : what does this structure imply on the form of labor supply functions ? and to what extend is it possible to recover the structural model from the observation of labor supply ?

2.2. The sharing rule

2.2.1. Definition

We first consider the decentralization of Pareto optima. The general result is the following :

Proposition 2.1. *Take any twice differentiable function $\bar{u}^2(w_1, w_2, y)$, and let $\bar{c}^1(w_1, w_2, y), \bar{c}^2(w_1, w_2, y), \bar{h}^1(w_1, w_2, y), \bar{h}^2(w_1, w_2, y)$ be the solutions of (2.1). Then there exists a sharing rule, i.e. two mappings $\rho^1(w_1, w_2, y)$ and $\rho^2(w_1, w_2, y)$, such that :*

- $\rho^1(w_1, w_2, y) + \rho^2(w_1, w_2, y) = y$

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- $\bar{c}^i(w_1, w_2, y)$ *and* $\bar{h}^i(w_1, w_2, y)$ *are solutions of :*

$$\max_{L^i, c^i} U^i \left[1 - h^i, c^i\right] \tag{2.2}$$

$$c^i = w_i . h^i + \rho^i(w_1, w_2, y)$$

$$0 \leq h^i \leq 1$$

Any decision process leading to Pareto-efficient outcomes can thus be equivalently interpreted as a two-stage process. At stage one, agents share non-labor income between them (of course, this choice may, and in general will, depend on the current values of wages and non labor income). At stage two, each member independently maximizes his (her) welfare, subject to the budget constraint stemming from stage one. In particular, member i's labor supply is of the form :

$$h^i(w_1, w_2, y) = H^i \left[w_i, \rho^i(w_1, w_2, y)\right]$$

where H^i is the Marshallian labor supply function associated to U^i.

The proof of Proposition 1 is straightforward : it is a particular case of standard results on the decentralizability of competitive equilibria[2]. Note, however, that in the previous statement, no assumption is made on the nature of the choice set for labor supplies; in particular, the result also applies to the case of discrete (participation) choices. However, the ρ^i may not be differentiable. We shall see below, in particular, that when, as it is the case here, the labor supply decision is dichotomous (i.e., $h^2 = 0$ or 1), then the continuity of $\bar{u}^2$ implies that, on the participation frontier, member 2's consumption c^2 is discontinuous, while his share of non labor income ρ^2 exhibits a kink.

In practice, to avoid any confusion, we shall explicitly distinguish between the sharing rule under (member 2's) participation, denoted ρ^2, and under non participation, denoted $\hat{\rho}^2$.

2.2.2. The links with 2's reservation utility

Let us first consider the case when member 2 does participate ($h^2 = 1$). His utility is thus $U^2 \left(c^2, 0\right)$, and we have that :

$$U^2 \left(c^2, 0\right) = U^2 \left(w_2 + \rho^2(w_1, w_2, y), 0\right) = \bar{u}^2(w_1, w_2, y)$$

[2]As remarked for instance by Apps and Rees (1997). For a precise proof, see Chiappori (1992).

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which can be inverted in :

$$\rho^2(w_1, w_2, y) = V^2\left[\bar{u}^2(w_1, w_2, y)\right] - w_2 = \Psi(w_1, w_2, y) - w_2$$

where V^2 is the inverse of the mapping $U^2(.,0)$, and Ψ is defined by :

$$\Psi(w_1, w_2, y) \doteq V^2\left[\bar{u}^2(w_1, w_2, y)\right]$$

Now, member 1's labour supply is of the form :

$$h^1(w_1, w_2, y) = H^1\left[w_1, y - \rho^2(w_1, w_2, y)\right]$$

A first consequence is that, for any $(w_1, w_2, y) \in P$:

$$\frac{-\rho^2_{w_2}}{1 - \rho^2_y} = \frac{h^1_{w_2}}{h^1_y} \doteq A(w_1, w_2, y) \tag{2.3}$$

Note that the function h^1, hence the ratio A, are empirically observable. Hence (2.9) provides a first characterization of ρ^2.

We now consider the non participation case ($h^2 = 0$).. Then 2's utility is $U^2(c^2, 1)$, and we have that :

$$U^2\left(c^2, 1\right) = U^2\left[\hat{\rho}^2(w_1, w_2, y), 1\right] = \bar{u}^2(w_1, w_2, y)$$

which can be inverted in :

$$\hat{\rho}^2(w_1, w_2, y) = W^2\left[\bar{u}^2(w_1, w_2, y)\right]$$

where W^2 is the inverse of the mapping $U^2(.,1)$. Since both V^2 and W^2 are increasing, we have that

$$\hat{\rho}^2(w_1, w_2, y) = F\left[\rho^2(w_1, w_2, y) + w_2\right] \tag{2.4}$$

for the increasing mapping $F = W^2 \circ (V^2)^{-1}$.

Just as above, member 1's labour supply is of the form :

$$h^1(w_1, w_2, y) = H^1\left[w_1, y - F\left(\rho^2(w_1, w_2, y) + w_2\right)\right]$$

and, for any $(w_1, w_2, y) \in N$:

$$\frac{-F'\left(\rho^2_{w_2} + 1\right)}{1 - F'\rho^2_y} = \frac{h^1_{w_2}}{h^1_y} \doteq B(w_1, w_2, y) \tag{2.5}$$

Note that the function A (respectively B) is defined only in the participation (respectively non participation) case.

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2.3. Participation decision

We now consider member 2's participation decisions. Remember, first, that member's 2 decision is discrete, whereas member 1 freely chooses her number of hours.

In what follows, we assume, for simplicity, that the corresponding participation frontier can be interpreted in terms of reservation wages; technically :

Assumption W : there exist a function $\gamma(w_1, y)$ such that :

$$h^2 = 1 \Leftrightarrow \omega_2 > \gamma(w_1, y)$$

Note that the function γ can be empirically observed, and will be considered as given in the remaining of this section. It follows that, if we disregard for the moments corner solutions for member 1, the space of the (w_1, w_2, y) can be partitioned into two areas :

- the participation set P, where $h^2 = 1$; this is equivalent to

$$\omega_2 > \gamma^2(w_1, y)$$

- the non participation set N, where $h^2 = 0$; this is equivalent to

$$\omega_2 < \gamma^2(w_1, y)$$

The frontier between P and N is called the participation frontier L, and is defined by the equation :

$$\omega_2 = \gamma^2(w_1, y)$$

Now, how will the frontier be characterized ? A basic result is the following :

Proposition 2.2. *At any $(w_1, w_2, y) \in L$, both members are indifferent between participating or not, taking into account the resulting change in their consumption share.*

Proof. That member 2 should be indifferent comes from the smoothness of $\bar{u}^2$ (see assumption R). Let us show that *member 1 must be indifferent as well.* Indeed, assume that he is not, so that his utility is discontinuous at some point

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K of L - say, it increases discontinuously if 2 participates. Take a point in the non participation set, arbitrarily close to K. Then a Pareto ameliorating change would be to increase 2's consumption by ε if he participates (this will be sufficient to induce him to participate, since he is indifferent, and 1's utility would then increase). But this would contradict the efficiency assumption. ■

When member 2 chooses to enter labor market, this will affect both members'welfare through three different channels. First, member 2's leisure will decrease; second, the household budget set extends; finally, the decision process will in general be modified, resulting in a change of the sharing rule. In particular, member 2's consumption must be discontinuous. Specifically, participation results in a discontinuous increase of member 2's consumtion, that exactly compensates the drop in leisure (so that that 2's utility is changed smoothly). How is this change determined ? The key point is that since member 1's number of hours is continuous, her consumption must also vary continuously. It follows that, on the participation frontier, *all additional income due to participation (i.e., w_2) goes to member 2.* Then the frontier is defined as the set of wage-income bundles such that a discontinuous increase of consumption equal to w_2 exactly compensates the drop in leisure resulting from participation. This is formally expressed by the following corollary :

Corollary 2.3. *At any point (w_1, w_2, y) on the participation frontier L, participation (locally) increase member 2's consumption by exactly w_2, and the frontier is characterized by the equation :*

$$\rho^2(w_1, \gamma(w_1, y), y) - \hat{\rho}^2(w_1, \gamma(w_1, y), y) = 0 \tag{2.6}$$

Using this result, we can rephrase the reservation wage assumption W in a more precise way. Specifically, W is a consequence of the following property :

Assumption W' :

$$\forall (w_1, w_2, y), \quad \left| [1 + \rho_{w_2}(w_1, w_2, y)] \cdot \left[1 - F'(\rho^2(w_1, z, y) + z)\right] \right| < 1 \tag{2.7}$$

To understand the meaning of assumption W', consider the increase in 2's consumption resulting from an infinitesimal increase dw_2 in 2's wage. Its magnitude

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depends on whether 2 is participating or not; for when 2 is participating, dw_2 increases both the household income and 2's bargaining power, while the first effect does not operate when 2 does not participate. Let dc^2 denote the consumption change in the former case, and $d\hat{c}^2$ in the latter. What (2.13) states is that the difference $dc^2 - d\hat{c}^2$ cannot be more than the initial increase dw_2.

Under (2.13), the mapping $z \mapsto \rho^2(w_1, z, y) - F\left(\rho^2(w_1, z, y) + z\right)$ is a contraction; then it can have only one fixed point for any given (w_1, y), which guarantees that γ is uniquely defined.

Using (2.4) and (2.6), we know that γ is characterized by the following equation :

$$\forall (w_1, y), \quad \rho^2(w_1, \gamma(w_1, y), y)) - F\left(\rho^2(w_1, \gamma(w_1, y), y) + \gamma(w_1, y)\right) = 0 \qquad (2.8)$$

This implies:

$$(\rho^2_{w_1} + \gamma_{w_1}\rho^2_{w_2}) = \frac{F'}{(1 - F')}\gamma_{w_1}$$

$$(\rho^2_y + \gamma_y \rho^2_{w_2}) = \frac{F'}{(1 - F')}\gamma_y$$

which can equivalently be written as :

$$\begin{aligned}(\rho^2_y + \gamma_y \rho^2_{w_2}) &= \frac{F'}{(1 - F')}\gamma_y \\ \rho^2_{w_1} &= \frac{\gamma_{w_1}}{\gamma_y}\rho^2_y\end{aligned}$$

all partials being taken on the frontier, i.e., at some point $(w_1, \gamma(w_1, y), y)$.

2.3.1. The extrapolation problem

The identification of the structural model from observed behavior will raise specific problems in some areas. Typically, while member 2's choice is discrete, member 1 is assumed to freely choose her working time. But, of course, the latter can be zero. Thus there will also be a participation frontier for member 1, which, again, is assumed to take the form of a reservation wage :

$$h^1 > 0 \Leftrightarrow w_1 > \delta\left(w_2, y\right)$$

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It follows that the plane is in fact divided into four areas, defined as follows (see Figure 1):

- P_1, where $h^1 > 0$ and $h^2 = 1$; this is equivalent to
$$\omega_1 > \delta(w_2, y) \text{ and } \omega_2 > \gamma(w_1, y)$$
- P_0, where $h^1 = 0$ and $h^2 = 1$; this is equivalent to
$$\omega_1 \leq \delta(w_2, y) \text{ and } \omega_2 > \gamma(w_1, y)$$
- N_1, where $h^1 > 0$ and $h^2 = 0$; this is equivalent to
$$\omega_1 > \delta(w_2, y) \text{ and } \omega_2 \leq \gamma(w_1, y)$$
- N_0, where $h^1 = 0$ and $h^2 = 0$; this is equivalent to
$$\omega_1 \leq \delta(w_2, y) \text{ and } \omega_2 \leq \gamma(w_1, y)$$

Of course,
$$P_1 \cup P_0 = P \text{ and } N_1 \cup N_0 = N$$

Now, although (as we shall see below) the individual utilitity functions and the sharing rule (or, equivalently, the function $\bar{u}^2$) can be recovered, up to an additive constant, from the observation of household behaviour in P_1 and N_1, in both P_0 and N_0, no behaviour can be observed at all, so nothing can be (independantly) identified. The standard solution, then, is to extrapolate to P_0 and N_0 the functions computed on P_1 and N_1. For instance, assume that we adopt a parametric approach, where the functional form of, say, U^1, U^2 and $\bar{u}^2$ are chosen a priori. The corresponding parameters (but one) can be identified from observations in P_1 and N_1; then, in principle, the structural model is known everywhere, although of course one has to assume that the functional form does not change over the space.

A nice feature of the 'extrapolation' technique, however, is that it is empirically testable. Indeed, we shall see that the sole observation of (member 1's) labor supply on P_1 and N_1 and of member 2's participation frontier γ allows to identify the structural model. Then member 1's participation frontier, δ, can be computed from the model, and compared to the actual frontier, as identified from the data; alternatively, one can simultaneously estimate labor supply and both participation

Include Figure 1 about here

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2.4. Identification of the structural model

We now describe the identification procedure on each of the four areas defined above. In H_{11}, this is straightforward, since the results of Chiappori (1992) apply. It follows that preferences and the sharing rule can locally be recovered, up to an additive constant.

We now consider the case of H_{10} (that of H_{01} is exactly similar). First, since member 2 does participate, his utility is $U^2(c^2, 0)$, and we have that :

$$U^2\left(c^2, 1\right) = \bar{u}^2(w_1, w_2, y)$$

which can be inverted in :

$$c^2 = V^2\left[\bar{u}^2(w_1, w_2, y)\right] = \rho^2(w_1, w_2, y)$$

where V^2 is the inverse of the mapping $U^2(., 0)$. Now, from the results above, member 1's labor supply is of the form :

$$h^1(w_1, w_2, y) = H^1\left[w_1, y - \rho^2(w_1, w_2, y)\right]$$

where H^1 is the Marshallian labor supply function associated to U^1.

A first consequence is that, for any $(w_1, w_2, y) \in P$:

$$\frac{-\rho^2_{w_2}}{1 - \rho^2_y} = \frac{h^1_{w_2}}{h^1_y} \doteq A(w_1, w_2, y) \tag{2.9}$$

where f_z stands for $\frac{\partial f}{\partial z}$. Note that the function h^1, hence the ratio A, are empirically observable. Hence (2.9) provides a first characterization of ρ^2.

Similarly, 2's labor supply takes the form :

$$h^2(w_1, w_2, y) = H^2\left[w_2, \rho^2(w_1, w_2, y)\right]$$

Since it vanishes on the frontier, we have that :

$$H^2\left[\gamma^2(w_1, y), \rho^2\left(w_1, \gamma^2(w_1, y), y\right)\right] = 0$$

We then use the following Lemma :

Lemma 2.4. *Take any point (w_1, w_2, y) on the participation frontier (i.e., such that $w_2 = \gamma^2(w_1, y)$), and assume that $\gamma^2_y \neq 0$. Then $\rho^2_y \neq 0$, and*

$$\frac{\gamma^2_{w_1}}{\gamma^2_y} = \frac{\rho^2_{w_1}}{\rho^2_y}$$

where all partials are taken at $(w_1, \gamma^2(w_1, y), y)$

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Proof. \$\$ ■

Let

\$\$\$

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We now consider the non participation case. Then 2's utility is $U^2(c^2, 1)$, and we have that :

$$U^2\left(c^2, 1\right) = \bar{u}^2(w_1, w_2, y)$$

which can be inverted in :

$$c^2 = W^2\left[\bar{u}^2(w_1, w_2, y)\right] = \hat{\Psi}(w_1, w_2, y)$$

where W^2 is the inverse of the mapping $U^2(., 1)$. Since both V^2 and W^2 are increasing, we have that

$$\hat{\Psi}(w_1, w_2, y) = F\left[\Psi(w_1, w_2, y)\right]$$

for the increasing mapping $F = W^2 \circ (V^2)^{-1}$; it follows that :

$$c^2 = F\left[\Psi(w_1, w_2, y)\right]$$

Now, member 1's program becomes :

$$\max_{L^1, C^1} U^1\left[1 - h^1, C^1\right] \tag{2.10}$$

$$C^1 = w_1.h^1 + y - F\left[\Psi(w_1, w_2, y)\right]$$
$$0 \leq h^1 \leq 1$$

This generates a labor supply of the form :

$$h^1(w_1, w_2, y) = H^1\left[w_1, y - F\left(\Psi(w_1, w_2, y)\right)\right]$$

and, for any $(w_1, w_2, y) \in N$:

$$\frac{-F'\Psi_{w_2}}{1 - F'\Psi_y} = \frac{h^1_{w_2}}{h^1_y} \doteq B(w_1, w_2, y) \tag{2.11}$$

Note that the function A (resp. B) is defined only on P (resp. N).

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2.5. Participation

Finally, the participation frontier L is defined by the fact that, at any $(w_1, w_2, y) \in L$, member 2 is indifferent between participating or not, taking into account the resulting change in his consumption share. A key remark is that *member 1 must be indifferent as well.* Indeed, assume that he is not, so that his utility is discontinuous at some point K of L - say, it increases discontinuously if 2 participates. Take a point in the non participation set, arbitrarily close to K. Then a Pareto ameliorating change would be to increase 2's consumption by ε if he participates (this will be sufficient to induce him to participate, since he is indifferent, and 1's utility would then increase). But this would contradict the efficiency assumption.

The interpretation is that when member 2 chooses to enter labor market, this will affect both members'welfare through three different channels. First, member 2's leisure will decrease; second, the household budget set extends; finally, the decision process will in general be modified, resulting in a change of the sharing rule. Note, also, that the latter must be discontinuous; it exactly compensates the drop in labor supply by member 2, in such a way that 2's utility is changed smoothly. Also, since member 1's number of hours is continuous, her consumption must also vary continuously. It follows that, on the participation frontier, *all additional income due to participation goes to member 1.* Formally, the frontier is thus defined by :

$$\forall (w_1, w_2, y) \in L, \quad \Psi(w_1, w_2, y) - F(\Psi(w_1, w_2, y)) = w_2 \tag{2.12}$$

To parametrize L, we choose to use a shadow wage condition; i.e., member 2 participates if and only if .

$$w_2 > \gamma(w_1, y)$$

for some γ, that describes the frontier. Note that this reservation wage property does not stem from the theoretical set-up as in standard labour supply models, but has to be assumed. Specifically, we assume the following :

Assumption :

$$\forall (w_1, w_2, y), \quad |\Psi_{w_2}(w_1, w_2, y). [1 - F'(\Psi(w_1, w_2, y)]| < 1 \tag{2.13}$$

In words : consider the increase in 2's consumption resulting from an infinetesimal increase dw_2 in 2's wage. Its magnitude depends on whether 2 is participating or not; for when 2 is participating, dw_2 increases both the household income and 2's bargaining power, while the first effect does not operate when 2 does not

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where the $\alpha^1, \alpha^2, \alpha^y, \beta$ are solutions to the following system (equivalent to (3.1)):

$$\begin{cases} -\alpha^2 + a\alpha^y = & a - 1 \\ -\alpha^2 + b\alpha^y - \dfrac{b}{\beta} = & 0 \\ \gamma_y \alpha^2 + \alpha^y - \frac{\gamma_y}{1-\beta} = & 0 \\ \alpha^1 - \frac{\gamma_{w_1}}{\gamma_y}\alpha^y = & 0 \end{cases} \tag{3.3}$$

where

$$\begin{aligned} a(w_1, y) &= A\left[w_1, \gamma(w_1, y), y\right] \\ b(w_1, y) &= B\left[w_1, \gamma(w_1, y), y\right] \end{aligned} \tag{3.4}$$

which implies :

$$\begin{aligned} a_{w_1} &= A_{w_1} + A_{w_2}\gamma_{w_1} \\ a_y &= A_y + A_{w_2}\gamma_y \\ b_{w_1} &= B_{w_1} + B_{w_2}\gamma_{w_1} \\ b_y &= B_y + B_{w_2}\gamma_y \end{aligned} \tag{3.5}$$

In particular, by differentiating (3.3), we get :

$$\begin{aligned} -\alpha^2_{w_1} + a\alpha^y_{w_1} &= a_{w_1}(1 - \alpha^y) \\ -\alpha^2_y + a\alpha^y_y &= a_y(1 - \alpha^y) \\ -\alpha^2_{w_1} + b\alpha^y_{w_1} + b\frac{\beta_{w_1}}{\beta^2} &= b_{w_1}(\frac{1}{\beta} - \alpha^y) \\ -\alpha^2_y + b\alpha^y_y + b\frac{\beta_y}{\beta^2} &= b_y(\frac{1}{\beta} - \alpha^y) \\ \gamma_y\alpha^2_{w_1} + \alpha^y_{w_1} - \frac{\beta_{w_1}\cdot\gamma_y}{(1-\beta)^2} &= \gamma_{w_1 y}(\frac{1}{1-\beta} - \alpha^2) \\ \gamma_y\alpha^2_y + \alpha^y_y - \frac{\beta_y\cdot\gamma_y}{(1-\beta)^2} &= \gamma_{yy}(\frac{1}{1-\beta} - \alpha^2) \\ \alpha^1_{w_1} - \frac{\gamma_{w_1}}{\gamma_y}\alpha^y_{w_1} &= \frac{\gamma_{w_1 w_1}\gamma_y - \gamma_{y w_1}\gamma_{w_1}}{(\gamma_y)^2}\alpha^y \\ \alpha^1_y - \frac{\gamma_{w_1}}{\gamma_y}\alpha^y_y &= \frac{\gamma_{y w_1}\gamma_y - \gamma_{yy}\gamma_{w_1}}{(\gamma_y)^2}\alpha^y \end{aligned} \tag{3.6}$$

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Note that, at any point, (3.3) allows to numerically compute the values of $\alpha^1, \alpha^2, \alpha^y, \beta$, while (3.6) gives the partials of these functions.

Now, (3.2) generates restrictions upon these partials. Specifically, the first three relations imply cross-derivatives conditions :

$$\alpha^1_y - \alpha^y_{w_1} = \gamma_y \alpha^2_{w_1} - \gamma_{w_1} \alpha^2_y \tag{3.7}$$

while the last one gives :

$$\frac{\beta_{w_1}}{\beta_y} = \frac{\alpha^1 + \alpha^2 \gamma_{w_1}}{\alpha^y + \alpha^2 \gamma_y} \tag{3.8}$$

In summary : assuming that A, B and γ have been recovered non parametrically, one should, at any point :

- compute a and b from (3.4), then the partials from (3.5)
- solve (3.3) for $\alpha^1, \alpha^2, \alpha^y, \beta$, then (3.6) for the partials
- check (3.7) and (3.8)

3.2. Outside the frontier

Let us start by the participation set P. The idea is clear : Ψ must satisfy the PDE (2.9), i.e.:

$$-\Psi_{w_2} + A\Psi_y = A - 1 \tag{3.9}$$

and, in addition, the boundary conditions are given by the values of the partials on the frontier. from standard theorems in PDE theory, this defines Ψ (up to an additive constant) provided the following condition is fulfilled. First, remark that, at any point on the frontier, (3.9) can be written as :

$$\Delta\Psi.\vec{u} = a - 1$$

where $\Delta\Psi$ denotes the gradient of Ψ, and $\vec{u}$ is the vector $(0, -1, a)'$. Now, the condition is that $\vec{u}$ is not tangent to the frontier L. Since the equation of L is :

$$w_2 - \gamma(w_1, y) = 0$$

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this condition states that, for all (w_1, y) :

$$a(w_1, y).\gamma_y(w_1, y) + 1 \neq 0 \tag{3.10}$$

If this relation is fulfilled on the frontier, then the PDE (2.9), together with the boundary condition, defines Ψ up to an additive constant.

Practically, there are cases where the PDE can be solved analytically. Then the solution is defined up to a function of 2 variables; and this function is identified by its values upon the frontier. The next section provides an example on a specific functional form.

But even when the PDE cannot be solved analytically, it is always possible to numerically compute Ψ using the PDE and the boundary condition on the frontier. One possible algorithm is the following :

1. Fix the value of Ψ on the frontier L (i.e., choose the constant, since the partials define Ψ up to a constant)

2. pick any point (w_1, w_2, y) on the frontier; then $\Psi(w_1, w_2, y)$ is known, and all the partials of Ψ at (w_1, w_2, y) are known as well

3. consider the point $(w_1, w_2 + dw_2, y)$; then

   $$\Psi(w_1, w_2 + dw_2, y) = \Psi(w_1, w_2, y) + \Psi_{w_2}(w_1, w_2, y).dw_2$$

   By doing this for all point on the frontier, we know the value of $\Psi(w_1, w_2, y)$ on the surface

   $$S = L + dw_2$$

4. compute the partials of Ψ on S. In the two directions tangent to S, this is implicitly done by the computation of Ψ on S; for the last one, just use the PDE (3.9) (note that this is possible because (3.10) holds true).

5. start from 1 using any point of S

Finally, in the non-participation set, the approach is exactly the same, using the PDE

$$-\Psi_{w_2} + B\Psi_y = \frac{B}{F'} \tag{3.11}$$

(remember that F has been exactly identified from the equations in the previous section). The sufficient condition then becomes : for all (w_1, y) :

$$b(w_1, y).\gamma_y(w_1, y) + 1 \neq 0$$

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4. A semi-parametric example

In this example, we make two assumptions on the functional forms. First, h^1 can be written as a function of w_1, on the one hand, and some function of y and w_2 that is quasi-linear with respect to y, on the other hand. Specifically, assume that h^1 is of the form :

$$h^1(w_1, w_2, y) = H^1[y + K(w_2), w_1]$$

where H^1 and K are arbitrary given functions (of course, these are estimated from observed labour supply). It follows that :

$$A = \frac{h^1_{w_2}}{h^1_y} = K'(w_2)$$

and the basic PDE (3.9) writes down :

$$-\Psi_{w_2} + K'(w_2)\Psi_y = K'(w_2) - 1$$

The general solution is of the form :

$$\Psi(w_1, w_2, y) = w_2 + y + \Gamma[y + K(w_2), w_1]$$

where Γ is a mapping to be identified.

The second functional assumption is that F is linear :

$$F(x) = f_0 + f_1 x$$

The second PDE (3.11) gives :

$$\Gamma_y[y + K(w_2), w_1] = \frac{1}{B - K'(w_2)}\left[1 + B\frac{1 - f_1}{f_1}\right] \qquad (4.1)$$

Since B can be identified (possibly non parametrically) from observation of labour supply of member 1, Γ_y is determined up to the constant f_1. Finally, on the frontier L, (2.14) writes down :

$$\gamma(w_1, y) + y + \Gamma(y + K(\gamma(w_1, y)), w_1) = \frac{\gamma(w_1, y) + f_0}{(1 - f_1)} \qquad (4.2)$$

Note that (4.2) strongly constrains the functional form of γ.

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ANNEXE 3

Modèle collectif d'offre de travail :

le cas d'offres continues

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Household Labor Supply, Sharing Rule and the Marriage Market *

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1. Introduction

Unitary versus non unitary approaches of household behavior Traditionally, household demand and labor supply decisions have been modeled as though household members were maximizing a unique, well behaved, utility function. Recent dissatisfaction with this 'unitary' model arose in a large part from the weakness of its theoretical foundations and its inability to be used to perform intra-household welfare analysis [Apps and Rees (1988)] or to study decisions such as marriage or divorce [Lundberg (1988)][1]. Since the standard unitary model considers the household as the basic unit decision, it is generally not possible to recover individual preferences or the parameters of the internal decision process that determines distribution of utilities and observed outcomes.

Alternative models have recently challenged the unitary model and attempted to incorporate different individual preferences and a model of intra-household decision making process. In particular, the Nash cooperative model assumes that a household maximizes the product of each member's utility in excess of a reservation level reflecting either marriage dissolution [*e.g.*, Manser and Brown (1980), McElroy and Horney (1981)] or an 'uncooperative' marriage [*e.g.*, Lundberg and Pollak (1993)]. A collective model of household labor supply has also been developed by Chiappori [(1988), (1992)]. His approach assumes that the intra-household decision process leads to Pareto efficient outcomes. In its most general form, the collective model nests Nash bargaining models as particular cases, since the latter are based on axioms that include Pareto efficiency. It also nests uncooperative repeated game models as long as they lead to Pareto efficient outcomes [see Lundberg and Pollak (1994) and Bergstrom (1997) for a discussion].

1.0.1. Empirical tests : price effects

A crucial issue concerns the design of empirical tests of these unitary and non-unitary theories. One set of testable predictions involves price and income effects. Existing results relate to the structure of the Slustky matrix corresponding to household demand. In a recent paper, Browning and Chiappori (1997) show that,

[1] An obvious exception is Becker's approach, where the household is explicitely modelled as a two-person decision unit, althought its behavior can be still analyzed with the tools of standard consumer theory.

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in the collective framework, this matrix need not be symmetric (as it would be the case in the unitary context), but that it can be written as the sum of a symmetric matrix and a matrix of rank at most one. This prediction is tested on Canadian consumption data; they find that, while symmetry is *not* rejected for subsamples of singles, it is rejected for couples, whereas the collective counterpart is not. This kind of test, however, requires the simultaneous estimation of a sufficiently large number of demand functions; moreover, it faces the standard difficulties of estimating price elasticities on cross sectional data.

Other predictions of the collective model have been derived in the case of labor supply behaviour, in particular by Chiappori (1992). An obvious advantage of this choice is that the estimation, on cross sections, of wage elasticities of labor supply is probably easier and more robust than that of price elasticities of consumption. Fortin and Lacroix (1997) have provided an empirical analysis based on a model that nests both the unitary and collective approaches. They found that while the restrictions of the unitary model are rejected, the collective restrictions are not. However the intra-household decision process is not very precisely estimated (only one parameter is significant). Also, while their sample includes households in which both spouses work, they do not correct for selectivity bias.

A first goal of the present paper is to extend the latter results. We build a collective model of household labor supply that generalizes the previous model by Chiappori (1992). We then estimate and test this model using 1989 PSID data for couples in which both spouses are working. Restrictions imposed by some alternative models are also tested. For estimation purposes, we use the Generalized Method of Moments (GMM) applied to a censored normal regression system with endogenous explanatory variables. Our approach takes into account the fact that female labor supply is censored at zero. Moreover, our estimators are robust to measurement error on the positive values of the censored dependent variable.

1.0.2. Empirical tests : income pooling and distribution factors

A second, and in principle simpler, consequence of the unitary model is pooling: only total exogenous income, not its distribution across household members, plays a role in the decision process. Using various measures of income, many studies attempted to test this income pooling hypothesis [*e.g.*, Schultz (1990), Thomas (1990, 1992), Phipps and Burton (1994)]. In all these studies, income pooling is strongly rejected. However, these tests raise two issues. First, while the re-

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sults reject the unitary model, they cannot be seen as supporting any alternative approach in particular since many models could explain observed results. In order to empirically support a specific model, one must derive, from this setting, restrictions that can potentially be, but are actually not falsified by empirical observations. Some recent work have attempted to derive and provide empirical tests of the collective approach [see Bourguignon *et al.* (1993), Browning *et al.* (1994), Browning and Chiappori (1994), Udry (1996)]. Moreover, a few studies tested different non-unitary models within a structural framework [Kapteyn and Kooreman (1991), Browning, Lechene and Rasheed (1996)]. However, these studies assume that labor supply is fixed, which may lead to specification problems. Moreover, individual preferences over consumption and leisure cannot be recovered.

The second issue concerns the use of personal incomes in the design of the tests. In most studies, the distribution of household (total or nonwage) incomes plays an important role in the derivation of tests. However this variable is particularly prone to reporting and measurement errors which may severely bias econometric tests [Chiappori *et al.* (1993), Lundberg and Pollak (1996)]. Moreover, endogeneity of earnings is likely to be a serious problem. Even in the case of nonwage incomes, one must expect these variables to be correlated with some unobservable individual characteristics that influence consumption and labor supply [Behrman, Pollak and Taubman (1995)]. For instance, variations in property income may reflect past saving and therefore unobserved (past and current) productivity heterogeneity[2].

It should however be emphasized that personal incomes (whether labor or non-labor) are not needed to perform tests of this kind. A more general idea is to use distribution factors [Bourguignon, Browning and Chiappori (1995)], that is, variables that can affect the decision process without influencing preferences or the joint consumption set. Typically, distribution factors include not only personal income variables, but also 'extra-environmental parameters' (EEPs) in McElroy's (1990) terminology, that affect opportunities of spouses outside marriage and can therefore influence respective bargaining powers and the final allocation. Like personal income variables, EEPs should have no effect on behavior in the unitary

[2]One way to avoid these problems is to exploit 'natural' experiments involving exogenous reallocation in intra-household incomes. Thus Lundberg, Pollak and Wales (1997) present a test of pooling based on a policy change in the United Kingdom that transferred child allowance from husbands to wives in the late 1970's. Unfortunately such natural experiments are very scarce.

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framework while the collective approach may restrict the form taken by their influence. Moreover, EEPs play a role in some cooperative models while they play none in others. Thus outside opportunities can influence behavior in the Manser-Brown/McElroy-Horney divorce threat bargaining model. However, they will have no effect in the Lundberg-Pollak 'separate spheres' bargaining model. In this case, the fallback option is not divorce but an uncooperative marriage in which spouses "revert to a division of work based on socially recognized and sanctioned gender roles".[3] In short, distirbution factors (or EEPs) represent a means of providing additional tests of the unitary and the collective models, and of discriminating between some non-unitary approaches.

A second contribution of the paper is the introduction, within a model of labor supply, of exogenous distribution factors that provide additional tests of the collective setting. The basic idea, here, is to use variables that proxy the situation on the marriage market. The intuition can be traced back to Becker (1991, ch.3), who emphasizes that the marriage market is an important determinant of intra-household utility distribution. In his approach, the state of the marriage market crucially depends on the sex ratio, that is, the relative supplies of males and females in the marriage market[4]. A nice theoretical foundation of this view has been provided by Rubinstein and Wolinsky (1985), who consider a non cooperative repeated game model of transactions between pairs of agents who meet randomly and bargain if they meet. Viewed as a model of marriage, it predicts that, following an increase in the number of males relative to females, males will be willing to concede a larger share of the gains from a marriage contract with a female [Bergstrom (1997)]. This discussion suggests that the sex ratio is a potentially important EEP. More recently, Grossbard-Shechtman (1993) has developed a simple general equilibrium model of the interactions between marriage markets and labor markets in which the sex ratio influences marriage and labor supply decisions. At the empirical level, using time series and cross-section data, she found that an increase in the sex ratio reduces the labor force participation of married women in U.S. However her econometric approach is not explicitly grounded in a structural model and uses aggregate data, which may create serious problems for

[3]Lundberg and Pollak (1993) develop a modified version of this model which allows for binding prenuptial agreements on a minimal transfer from one spouse to the other. This model could allow EEPs to influence distribution within marriage in the long run by influencing the value of this transfer in new marriages.

[4]Lundberg and Pollak (1996) insist on the specific features of the marriage contract (*e.g.*, whether marriage agreements are binding or not) as another determinant of the marriage market. Unfortunately, it is difficult to construct empirical measures of these features.

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empirical testing.

In this paper, we introduce a sex ratio variable in our parametric model of household labor supply and derive the restrictions imposed by the collective model when allowing for this variable. In this setting, household decisions can be modeled as a two-step process. First, the individuals share their total nonwage income according to a sharing rule; then they maximize their own utilities subject to separate budget constraints. The sex ratio influences labor supply via this sharing rule. Also, from observable labor supply behavior, it is possible to recover individual preferences (up to a translation) and the sharing rule (up to an additive constant).

Our main conclusions are as follows :

- the parametric restrictions implied by the collective model are not rejected, although they are the verge of rejection in one version of the model (and clearly not rejected in the other version).
- the estimation of the sharing rule suggest small but significant effects of wages upon intrahousehold allocation. For instance, we find that a one dollar increase in male (resp. female) hourly wage - which, given the average number of hours per year, represents an average additional yearly income above $2,000 (resp. above $1,500) - increases (resp. decreases) the male's share of non labor income by $470 (resp. $85).
- the sex ratio does influence labor supply behavior in a significant way. Moreover, the directions are those predicted by theory. Other things equal, an increase in the proportion of women raises female labor supply while it reduces male labor supply; this suggests an effect of the sex ratio on intrahousehold income distribution that, according to our estimates, is around $280 a year per percentage point. Even more interesting is the fact that no such effect can be observed on singles; this indicates that, although the sex ratio may partly reflect conditions on the labor market, it probably is not the whole story.

2. The Model

In this section, we develop a collective labor supply model which extends Chiappori's (1992) approach to take into account distribution factors. In this framework, the household consists of two individuals with distinct utility functions

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and the decision process, whatever its true nature, leads to Pareto-efficient outcomes. This assumption seems quite natural, given that spouses usually know each other's preferences pretty well (at least, after a certain period of time) and intereact very often. Therefore, they are unlikely to leave Pareto-improving decisions unexploited [however, see Udry (1996)].

Formally, let h^i and C^i denote respectively member i's labor supply (with $0 \leq h^i \leq 1$) and consumption of a private hicksian composite good whose price is set to unity, for $i = 1, 2$. Household members have egoistic preferences[5] represented by strictly quasi-concave and increasing, continuously differentiable, utility functions $U^i(1 - h^i, C^i, z)$, where z is a $K-$vector of preference factors, such as age and education of the two agents, number and age of children, *etc.* For convenience, the vector z is assumed the same in both utility functions. Also, let w_1, w_2, y denote respective wage rates and household nonwage income. Finally, s denotes the $L-$vector of distribution factors.

Under the collective framework, intra-household decisions are Pareto-efficient, that is, for given (w^1, w^2, y, z, s), there exists a constrained utility $\bar{u}^2(w^1, w^2, y, z, s)$ such that the (h^i, C^i) solve the following program:

$$\max_{\{h^1,h^2,C^1,C^2\}} U^1 = U^1(1 - h^1, C^1, z)$$

$$\text{subject to} \qquad (\bar{P})$$

$$\begin{aligned} U^2(1 - h^2, C^2, z) &\geq \bar{u}^2(w_1, w_2, y, z, s), \\ w_1 h^1 + w_2 h^2 + y &\geq C^1 + C^2, \\ 0 &\leq h^i \leq 1, \quad i = 1, 2, \end{aligned}$$

where the function $\bar{u}^2$ is assumed continuously differentiable in its arguments. It should thus be clear that the particular location of the solution on the Pareto frontier depends on all relevant parameters, since the value of $\bar{u}^2$ depends on w_1, w_2, y, z and s. Moreover, since the vector of distribution factors, s, appears only in $\bar{u}^2$, a change in s does not affect the Pareto frontier but only the final outcome on it. In the particular case where $\bar{u}^2$ is a assumed to be constant,

[5]As shown below, the model also holds in the more general case of 'caring' preferences [see Becker (1991)].

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the collective framework corresponds to the unitary model. In this situation, the distribution factors have no effect on behavior.

As far as efficiency is concerned, there is basically no restriction upon the form of the function $\bar{u}^2$. However, for the estimation process, we shall assume that $\bar{u}^2$ is monotonic. A natural interpretation is that an increase in w_1, w_2 or y should presumably increase both member's utility, whereas, in the line of, say, a bargaining interpretation of the decision process, any distribution factor that is positively (resp. negatively) correlated with member 2's (resp. 1's) threat point should increase $\bar{u}^2$.

Now, it is well known, from the second fundamental welfare theorem, that Pareto optima can be decentralized in an economy of this kind [see Chiappori (1992)]. Specifically, we have the following result :

Proposition 1. *The program $(\bar{P})$ is equivalent to the existence of some function $\phi(w_1, w_2, y, z, s)$ such that each member i $(i = 1, 2,)$ solves the program :*

$$\max_{\{h^i, C^i\}} U^i(1 - h^i, C^i, z)$$

$$\text{subject to} \qquad (\bar{P}')$$

$$\begin{aligned} w_i h^i + \phi^i &\geq C^i \\ 0 &\leq h^i \leq 1 \end{aligned}$$

where $\phi^1 = \phi$ and $\phi^2 = y - \phi$.

Proof. This is an immediate consequence of the second welfare theorem. ■

The interpretation is that the decision process can always be considered as a two stage process : first, nonwage income is allocated between household members and then, each member separately chooses labor supply (and private consumption), subject to the corresponding budget constraint.[6] The function ϕ is called the *sharing rule*; it describes the way non labor income is split, as a function of wages, non labor income, distribution factors and other observables characteristics.

[6] In the presence of household public goods, a sharing rule can be defined but conditionally on the level of these public goods.

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2.1. Restrictions on Labor Supplies and the Sharing Rule

The collective framework imposes certain restrictions on the labor supply functions. To analyze this issue, let us first define the unrestricted labor supply functions:

$$h^1 = h^1(w_1, w_2, y, z, s), \tag{2.1}$$
$$h^2 = h^2(w_1, w_2, y, z, s). \tag{2.2}$$

In the remainder, we assume these functions are continuously differentiable. From $(\bar{P}')$, and assuming an interior solution, labor supply functions can be written as:

$$h^1 = H^1(w_1, \phi(w_1, w_2, y, z, s), z) \tag{2.3}$$
$$h^2 = H^2(w_2, y - \phi(w_1, w_2, y, z, s), z) \tag{2.4}$$

The particular structure of equations (2.3) and (2.4) imposes testable restrictions on labor supply behavior; furthermore, it is possible to recover the partials of the sharing rule. The key point is that each member's wage rate can only have an income effect on his or her spouse's behavior through its effect on the sharing rule. Also, distribution factors and nonwage income influence behavior through the sharing rule. Consequently, information on the impact of these variables on labor supply behavior of member 1 allows us to estimate the marginal rate of substitution between w_2 and y as well as between w_2 and each s_l in the sharing rule. The same argument applies to member 2's behavior; and the two nonwage shares must add up to one. This will allow to exactly recover the partial derivatives of the sharing rule. Finally, the equality of the effect of each distribution factor on the sharing rule as computed from both labor supply functions yields L additional restrictions.

To be more precise, using equations (2.3) and (2.4), let us define $A = h^1_{w_2}/h^1_y$, $B = h^2_{w_1}/h^2_y$, $C_l = h^1_{s_l}/h^1_y$ and $D_l = h^2_{s_l}/h^2_y$, whenever $h^1_y \cdot h^2_y \neq 0$, for $l = 1, \cdots, L$. Note that all these variables are observable and can thus be estimated. The following results hold:

Proposition 2. *Take any point such that* $h^1_y \cdot h^2_y \neq 0$. *Then*

(i) If there exists exactly one distribution factor, and that it is such that $C \neq D$, *the following conditions are necessary for any pair* (h^1, h^2) *to be solutions of* $(\bar{P}')$ *for some sharing rule* ϕ:

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(a)

$$\frac{\partial}{\partial s}\left(\frac{D}{D-C}\right)=\frac{\partial}{\partial y}\left(\frac{CD}{D-C}\right)$$

(b)

$$\frac{\partial}{\partial w_1}\left(\frac{D}{D-C}\right)=\frac{\partial}{\partial y}\left(\frac{BC}{D-C}\right)$$

(c)

$$\frac{\partial}{\partial w_2}\left(\frac{D}{D-C}\right)=\frac{\partial}{\partial y}\left(\frac{AD}{D-C}\right)$$

(d)

$$\frac{\partial}{\partial w_1}\left(\frac{CD}{D-C}\right)=\frac{\partial}{\partial s}\left(\frac{BC}{D-C}\right)$$

(e)

$$\frac{\partial}{\partial w_2}\left(\frac{CD}{D-C}\right)=\frac{\partial}{\partial s}\left(\frac{AD}{D-C}\right)$$

(f)

$$\frac{\partial}{\partial w_2}\left(\frac{BC}{D-C}\right)=\frac{\partial}{\partial w_1}\left(\frac{AD}{D-C}\right)$$

(ii) assuming that conditions (a) – (f) hold and for a given z, the sharing rule is defined up to an additive constant κ and its partial derivatives are given by

$$\phi_y=\frac{D}{D-C}$$
$$\phi_s=\frac{CD}{D-C}$$
$$\phi_{w_1}=\frac{BC}{D-C}$$
$$\phi_{w_2}=\frac{AD}{D-C}$$

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(ii) Finally, if there are several distribution factors ($l = 1, \cdots, L$), an additional set of necessary and sufficient conditions are :

$$\frac{C_l}{D_l} = \frac{C_1}{D_1}\ ,\ \ l = 2, \cdots, L$$

Demonstration: see Appendix

Conditions $(a)-(f)$ are analogous to Slutsky restrictions in the (general) sense that they provide a set of partial differential equations and inequalities that must be satisfied by the labor supply equations (2.1) and (2.2) in order to be consistent with the collective model. Note, however, that their form is quite different from those obtained in Chiappori (1992) for a similar model without distribution factor. As a matter of fact, the introduction of distribution factors deeply changes the way the model is identified. In Chiappori's initial contribution, identification required second order derivatives. In our case, on the contrary, *first order derivatives are enough.* This suggests that the kind of identification that may obtain is more robust in this case. Note, however, that an alternative approach using second derivatives can still be used (in the case, for instance, when $C_l = D_l$ for all l); it leads to identical results.

Interestingly, the model implies that the relative effects of distribution factors on each labor supply are equal, that is, $h^1_{s_l}/h^1_{s_1} = h^2_{s_l}/h^2_{s_1}$, for $l = 2, \cdots, L$, since both members of this equation are equal to ϕ_{s_l}/ϕ_1. [7] These latter restrictions are very general in the sense that they can be shown to hold also when public goods and externalities are allowed [see Bourguignon, Browning and Chiappori (1995)]. The basic reason is that, as mentioned earlier, distribution factors affect consumption and labor supply choices only through the allocation chosen on this frontier -or equivalently, through the respective weighting of each spouse's utility implicit in this allocation. Since this weighting is unidimensional, this implies that the ratio of the impacts of all distribution factors on the two labor supplies are equal. It is worth stressing that these restrictions appear only when there are at least two distribution factors.

2.2. Caring

Up to now, we have assumed that preferences are egoistic. However, as shown in Chiappori (1992), all the previous results also hold in the more general case of

[7] These restrictions are implied by conditions (c), since, from the latter, one has $C_1/C_l = D_1/D_l$, and since, from the definitions of the C's and D's, one has $h^1_{s_l}/h^1_{s_1} = C_1/C_l$ and $h^2_{s_l}/h^2_{s_1} = D_1/D_l$, for $l = 2, \cdots, L$.

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"*caring*" agents [see Becker (1991)], that is, whose preferences are represented by a utility function that depends on both his or her egoistic utility and his or her spouse's. Formally, the member i's utility function can be written as:

$$W^i = W^i[U^1(1-h^1, C^1, z),\ U^2(1-h^2, C^2, z)], \quad \text{for} \quad i = 1, 2. \tag{2.5}$$

W^i is continuous, increasing and quasi-concave in "egoistic" utilities U^1 and U^2. These utility functions impose separability between a member's own private goods and his or her spouse's. It is clear that any decision that is Pareto efficient under caring hypothesis would also be Pareto efficient, were the agents egoistic. Assume not; then it would be possible to increase the egoistic utility of a member without decreasing the utility of the other. But this would increase the caring utility of at least one member without reducing the caring utility of any member, a contradiction. In fact, the Pareto frontier of caring agents is a subset of the Pareto frontier derived by assuming assumed that they are egoistic [Chiappori (1992)].

In the next section, we will use these results to derive the parametric restrictions imposed by the collective model to the particular labor supply system considered in our econometric approach, and to recover the corresponding sharing rule.

3. Parametric Specification of Labor Supplies

In order to estimate and test a collective model of labor supply, we must first specify a functional form for individual labor supply functions. The following unrestricted system:

$$\begin{aligned} h_1 &= f_0 + f_1 \log w_1 + f_2 \log w_2 + f_3 y + \\ &\quad f_4 \log w_1 \log w_2 + f_5 s + f_6' z + f_7' s z; \end{aligned} \tag{3.1}$$

$$\begin{aligned} h_2 &= m_0 + m_1 \log w_1 + m_2 \log w_2 + m_3 y + \\ &\quad m_4 \log w_1 \log w_2 + m_5 s + m_6' z + m_7' s z, \end{aligned} \tag{3.2}$$

where the f_i's and the m_i's, for $i = 1, \cdots, 5$, are scalar, and the f_i's and the m_i's, for $i = 6, 7$, are $K-$vectors of parameters. For convenience and to reflect the empirical section, it is assumed that the vector of distribution factors, s, has only one element ($i.e.$, $L = 1$).

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The generalized semi-log system (3.1) and (3.2) satisfies a number of desirable properties. First, in its unrestricted form, it does not impose all the (equality) conditions of the collective model. Therefore, the latter yields a set of restrictions that can be empirically tested. Second, as shown below, these restrictions do not impose unrealistic constraints on behavior. Third, assuming that the collective restrictions are satisfied, it is possible to recover a closed form for the sharing rule (up to an additive constant κ) and for the pair of individual indirect utility functions (for any given κ). Finally, the fact that equations (3.1) and (3.2) are linear in parameters eases the estimation, specially when corner solutions have to be taken into account.

Of course, this generalized semi-log system also has some limitations. While some restrictions of the unitary model consistent with this system do not impose unrealistic labor supply on behavior, other restrictions do and therefore cannot be tested.[8] However, this should not be a serious problem since the unitary model of household labor supply has been rejected in many empirical work [*e.g.*, Kooreman and Kapteyn (1986), Lundberg (1988), Fortin and Lacroix (1997)]. Second, labor supply curves are either everywhere upward sloping or everywhere backward bending, though the sign of $\partial h^i/\partial w_i$ can change with the level of w_j $(j \neq i)$. Note, however, that the log form for the wage rates is likely to reflect more realistic behavior than the linear form that is frequently used in empirical studies [*e.g.*, Hausman (1981), Triest (1990)]. Thus it allows the effect of the wage rate on labor supply to decrease with the level of hours of work (when the labor supply is upward sloping), which is likely to be the case. It is also worth mentioning that our specification allows for interactions between distribution and preferences factors.

The restrictions imposed by the collective model (see Proposition 1) to the generalized semi-log system can easily be derived. First, using the definitions of

[8] More specifically, the unitary model imposes that labor supplies are independent from any distribution factor and that the Slutsky matrix of compensated wage effects is symmetric and semi-definite positive. The former constraint requires that $f_5 = f_{7k} = m_5 = m_{7k} = 0$, for all k. These restrictions can be tested. However, the symmetry of the Slutsky matrix requires in addition either that (i) $f_2 = f_3 = f_4 = m_1 = m_3 = m_4 = 0$, which implies that each labor supply depends only on own wage rate and on preference factors, or that (ii) $f_0 = m_0, f_3 = m_3, f_{6k} = m_{6k}$ and $f_1 = f_2 = f_4 = m_1 = m_2 = m_4 = 0$, for all k, which implies that labor supplies are the same and depend only on nonwage income and on preference factors. It is clear that these two cases impose severe constraints on behavior.

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$A - D_1$ and of $\alpha - \beta$, one gets:

$$A = \frac{f_2 + f_4 \log w_1}{w_2 f_3}, \qquad B = \frac{m_1 + m_4 \log w_2}{w_1 m_3},$$
$$C = \frac{f_5 + f_7' z}{f_3}, \qquad D = \frac{m_5 + m_7' z}{m_3}$$

The condition $C \neq D$ is satisfied unless

$$\frac{m_3}{f_3} = \frac{m_5}{f_5} = \frac{m_{71}}{f_{71}} = \ldots = \frac{m_{7K}}{f_{7K}} = T$$

Then the necessary and sufficient conditions take the following form ::

$$m_4 \left(f_5 + f_7' z\right) = f_4 \left(m_5 + m_7' z\right)$$

Since this condition must hold for any z, this imposes that $m_4 f_5 = f_4 m_5$ and $m_4 f_{7k} = f_4 m_{7k}$, for $k = 1, \cdots, K$, or, equivalently (assuming that no parameter is equal to zero)

$$\frac{m_4}{f_4} = \frac{m_5}{f_5} = \frac{m_{71}}{f_{71}} = \ldots = \frac{m_{7K}}{f_{7K}} = T$$

If the latter are satisfied, then the partials of ϕ are given by :

$$\begin{aligned}
\phi_y &= \frac{f_3 m_4}{\Delta} \\
\phi_s &= \frac{m_4}{\Delta}\left(f_5 + f_7' z\right) \\
\phi_{w_1} &= \frac{f_4}{\Delta}\frac{m_1 + m_4 \log w_2}{w_1} \\
\phi_{w_2} &= \frac{m_4}{\Delta}\frac{f_2 + f_4 \log w_1}{w_2}
\end{aligned}$$

where $\Delta = f_3 m_4 - f_4 m_3$.

$$\frac{m_4}{f_4} = \frac{m_5}{f_5} = \frac{m_{71}}{f_{71}} = \cdots = \frac{m_{7K}}{f_{7K}}. \tag{3.3}$$

Equations (3.3) impose testable cross-equation restrictions in our labor supply system. They require the ratio of the marginal effects of the cross term in $\log w_1$ and $\log w_2$ to be equal to the corresponding ratio of the marginal effects of the distribution factor on labor supplies. It should be stressed that our collective

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model imposes no (equality) restriction when there is no distribution factor or no cross term as $\log w_1 \log w_2$. Also, the inequalities conditions (d) and (e) must be verified for the collective model to hold.

Now, assuming that conditions $(a)-(e)$ are satisfied, one can easily recover the sharing rule. Solving the four differential equations system of partial derivatives of the sharing rule, as given in (ii) of Proposition 1, one obtains:

$$\begin{aligned}\phi &= \frac{1}{\Delta}\left[m_1 f_4 \log w_1 + f_2 m_4 \log w_2 + f_4 m_4 \log w_1 \log w_2 + \right.\\ &\quad \left. f_3 m_4 y + m_4 f_5 s + m_4 f_7' z s\right] + \xi(z), \qquad (3.4)\end{aligned}$$

In equation (3.4), the function $\xi(z)$ is not identifiable, since the variable z affects both the sharing rule and the preferences. This reflects the fact that the sharing rule can be recovered up to an additive constant for each individual.

4. Econometric Issues

It is natural to estimate restricted or unrestricted versions of the labor supply system (3.1) and (3.2) using an instrumental variable (IV) approach, given that the wage rates and nonwage income variables are potentially endogenous [see Mroz (1987)]. However, one must take into account the fact female labor supply, h^1, is censored at zero. The problem is much less important in the case of male labor supply since less than 5% of our male sample did not work over the period considered. Consider the following latent variable model for the household i:

$$h_i^{1*} = \delta_1' x_{i1} + \mu_{i1}, \qquad (4.1)$$
$$h_i^2 = \delta_2' x_{i2} + \mu_{i2}, \qquad (4.2)$$

where the δ_j's are the vectors of parameters to be estimated, the x_{ij}'s are the vectors of explanatory variables, some of which are potentially endogenous and the μ_{ij}'s are error terms with $(\mu_{i1}, \mu_{i2}) \sim N(0, \Sigma)$, for $j = 1, 2$. The variable h_i^{1*} is a latent variable for the female i's labor supply and the variable h_i^2 is the male i's observed labor supply (no censoring at zero is assumed). While the value of h_i^{1*} is not observable, an indicator function h_i^1 is observed and is related to h_i^{1*} in the following manner:

$$h_i^1 = \begin{cases} h_i^{1*} & \text{if } h_i^{1*} > 0 \\ 0 & \text{otherwise.} \end{cases}$$

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Using GMM to estimate the system (4.1) and (4.2) with one latent equation is motivated by the fact that the standard tobit model can be estimated consistently using a Nonlinear Least Squares (NLLS) method [Amemyia (1985)]. Two general classes of estimators are available for this purpose: estimators based on either the expectation function (EF) of h_i^j or the expectation function of h_i^j conditional on $h_i^1 > 0$ (CEF), for $j = 1, 2$. Moreover, as shown by Stapleton and Young (1984), the EF and the CEF expectation estimators are robust to measurement errors on the positive value of the censored dependent variable, which is not the case for the standard maximum likelihood (ML) estimator of the system of equations.[9]

Since wage rates are not observable for females who do not work, it is natural to focus on the CEF estimator. Using properties of the truncated multivariate normal distribution, it is easy to show that the CEFs of the female and male labor supplies, given x_{i1} and x_{i2}, are respectively given by:

$$E(h_i^1 \mid x_{i1}, h_i^1 > 0) = \delta_1' x_{i1} + \sigma_1 \frac{\phi(\cdot)}{\Phi(\cdot)} \equiv h^1(x_{i1}, \delta_1^+) \quad (4.3)$$

$$E(h_i^2 \mid x_{i2}, h_i^1 > 0) = \delta_2' x_{i2} + \sigma_{12}/\sigma_2 \frac{\phi(\cdot)}{\Phi(\cdot)} \equiv h^2(x_{i2}, \delta_2^+), \quad (4.4)$$

where $\phi(\cdot)$ and $\Phi(\cdot)$ are respectively the density and the cumulative functions of the standard normal distribution evaluated at $\delta_1' x_{i1}/\sigma_1$ and where $\delta_1^+ = (\delta_1', \sigma_1)'$ and $\delta_2^+ = (\delta_2', \sigma_1, \sigma_{12}/\sigma_2)'$. The expression $\phi(\cdot)/\Phi(\cdot)$ is the inverse of Mills' ratio. In the presence of this expression, the CEF of each h_i^j is a nonlinear function of δ_j^+. Using equations (4.3) and (4.4), the female and male labor supply equations, conditional on $h_i^1 > 0$, can be written as:

$$h_i^1 = h^1(x_{i1}, \delta_1^+) + \epsilon_i^1, \quad (4.5)$$

$$h_i^2 = h^2(x_{i2}, \delta_2^+) + \epsilon_i^2, \quad (4.6)$$

[9]The reason why the standard ML estimator is not consistent in the presence of measurement error is that the estimator of one of the parameters, the variance of the error term of the censored equation, is based in part on the variance of the corresponding dependent variable. Since measurement error increases this variance, its estimator is biased upward and this bias is transmitted to the coefficient estimators of the equation system. On the contrary, measurement error does not affect the consistency of EF or CEF estimators since no parameters from the (unspecified) distribution of the measurement appear in the EF or in the CEF. Of course, these estimators are asymptotically less efficient than a correctly specified ML estimator, but the latter requires the specification of the measurement error.

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where, by equations (4.3) and (4.4), $E(\epsilon_i^j \mid x_{ij}) = 0$, for $j = 1, 2$.

Naturally, the labor supply system (4.5) and (4.6) could be estimated using an NLLS approach if the x_{ij} variables were all exogenous. However, it is inappropriate if any of them is endogenous. We therefore rely on GMM estimates with White's correction for heteroskedasticity. More specifically, define $h^j(x_j, \delta_j^+)$ as the mean function of the vector of observations on labor supply, h^j, where x_j is the corresponding matrix of observations on explanatory variables. Denote $e = [(h^1 - h^1(x_1, \delta_1^+))', (h^2 - h^2(x_2, \delta_2^+)]'$ as the stacked vector of error terms. Assume there exists a set of instruments W and define $\bar{W} = I \otimes W$, where I is a 2×2 identity matrix. The moment conditions $E(e_i \bar{W}_i) = 0$ are assumed to be satisfied. The GMM estimator for $\delta^+ (= (\delta_1^+, \delta_2^+))$, in the case where there are more moment conditions than there are parameters to estimate, is chosen to minimize:

$$e'[\bar{W}(\bar{W}'\Omega\bar{W})^{-1}\bar{W}']e, \tag{4.7}$$

where $(\bar{W}'\Omega\bar{W})$ is the covariance matrix of $\bar{W}'e$. The diagonal matrix Ω is not observable. However, it can be approximated by a heteroskedasticity-consistent covariance matrix estimator (HCCME) $\hat{\Omega}$, the i^{th} diagonal element of which being given by $\hat{\epsilon}_i^2 = [h_i^1 - h^1(x_{i1}, \hat{\delta}_1^+), h_i^2 - h^2(x_{i2}, \hat{\delta}_2^+]^2$, where the estimated parameters are obtained in a first step using a nonlinear instrumental estimation. The latter consists in minimizing equation (4.7) with Ω being replaced by the identity matrix [see White (1980)].

While the estimator of δ^+ obtained using the HCCME $\hat{\Omega}$ has good asymptotic properties, it is possible to modify it so that it has better finite-sample properties. Mackinnon and White (1985) concluded that a different HCCME (denoted $\tilde{\Omega}$) generally seems to perform better in Monte Carlo experiments. The i^{th} diagonal element of this HCCME is given by $\hat{\epsilon}_i^{\,1}/(1 - \hat{h}_i)^1$, where $\hat{h}_i = f_i(f'f)^{-1}f_i'$ is the i^{th} diagonal of the "hat" matrix $\hat{P}_f$ that projects orthogonally onto the space spanned by the columns of f, where f is the stacked matrix of derivatives of $h^1(x_1, \delta_1^+)$ and of $h^2(x_2, \delta_2^+)$ with respect to the parameters of the model for all observations.

Following an iteration process suggested by Mackinnon and Davidson (1993), equation (4.7) is first minimized starting with $\Omega = I$; next we compute $\tilde{\Omega}$ and minimize (4.7) again. Upon convergence, we compute $\tilde{\Omega}$ with the new set of parameter estimates and minimize again (4.7) until both $\tilde{\Omega}$ and $\tilde{\delta}^+$ converge to stable values. The covariance matrix of $\tilde{\delta}^+$ is then estimated as:[10]

$$(\tilde{f})'\bar{W}(\bar{W}'\tilde{\Omega}\bar{W})^{-1}\bar{W}'(\tilde{f}). \tag{4.8}$$

[10]See Greene (1993), p.379.

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where $\tilde{f}$ corresponds to the matrix f evaluated at $(\tilde{\delta}^{+})$.

It is worth mentioning that our approach does not allow us to identify the standard deviation of the error term associated with the male labor supply equation, σ_2; only σ_{12}/σ_2 is identifiable (see equation (4.4)). Also, since σ_1 is identifiable from equation (4.3), the correlation coefficient between the error terms of male and female labor supplies, ρ $(= \sigma_{12}/\sigma_1\sigma_2)$, is identifiable.

5. Data and empirical results

5.1. Data

The data we use in this study come from the University of Michigan Panel Study of Income Dynamics (PSID) for the year 1988 (interview year 1989). Our sample consists of 2034 households in which both spouses have positive hours of work and whom are between 18 and 60 years of age. First-year cohabitors are excluded from the sample in order to concentrate on 'stable' households.

The dependent variables, male and female annual hours of work, are defined as total hours of work on all jobs during 1988. The measure of the wage rate is the average hourly earning, defined by dividing total labor income over annual hours of work. Nonwage Income includes total transfers (from government and relatives) and income from interest and dividends.[11] Our sex-ratio index is computed at the State level using data from the Census of Population and Housing of 1990. It corresponds to the number of males that are of the same age and same race as the husband of each household over the number of males and females that are of the same race and age group.[12]

Table 1 presents the sample characteristics. The upper panel concerns variables that are treated as endogenous in the ensuing empirical analysis. The lower panel concerns variables that are either used as controls or as instruments. As expected, men work on average more yearly hours than women and earn a somewhat higher hourly wage rate. The large standard deviation on nonwage income is primarily due to the fact that many young households earn little, if any, income from dividends and/or interests. The next two lines report the average number of preschoolers and school age children per household. These two variables are

[11] Naturally, nonwage income is treated has an endogenous variable in the empirical section.

[12] We experimented with various definitions of the sex-ratio: means of sex-ratios using the number of females who are two years younger than the husband or based on individuals who are at most 2 or 5 years younger than the husband of each household. The results were very robust to the definition used.

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treated as endogenous in the empirical section of the paper. Although there is mixed evidence concerning the endogeneity of women's labor supply and number of children [see, *e.g.*, Schultz (1980) and Mroz (1987)], we deem preferable to instrument these variables.[13]

Men are also slightly older than their spouse but they both have similar schooling, which in turn is above that of their respective father. The distribution by race is identical among men and women. A closer look at the data reveals that there are very few interracial marriages in our sample. The sex ratio is thus computed on the assumption that the relevant marriage market is limited to one's own race. The next lines report the mean of the sex ratios for Whites, Blacks and Others (essentially Asians and American Indians). Although the means are relatively similar, the sex ratios for Blacks and Others depict much more variation. This increased variation is observable both state-wide and age-wise.

5.2. Empirical Results

The parametric form that will be estimated was presented in equations (3.1) and (3.2). Recall that the vector of distribution factors, s, appears twice in each labor supply functions, once by itself and once in interaction with the preference factors. As mentioned earlier, given the difficulty of obtaining information on distribution factors, we only consider one of those, namely the sex ratio.[14] After considerable investigation, only an interactive term between the sex ratio and a dummy variable that indicates whether there are any preschool children in the household, has been introduced in the model. In doing so, we wish to investigate whether the impact of the sex ratio is hindered by the presence of preschoolers. Indeed, one could argue that the effect of alternative options in the marriage market on behavior of spouses (specially women) is likely to be much less effective in the case of couples with young children. Finally, the other preference factors are the number of school age children, education, age, and dummy regional variables. This specification is relatively standard in the labor supply literature [*e.g.* Mroz (1987), Smith Conway (1997)].

Before dwelling into the results, the issue of endogenous covariates must be addressed. Indeed, unobserved individual characteristics may be positively corre-

[13] As will see later, most results are unaffected by instrumenting these variables.

[14] We did estimate the model by distinguishing between husband's and wife's nonwage income to provide one additional distribution factor. Unfortunately, the parameter estimates were never statistically significant when doing so. One basic difficulty lies in properly defining and measuring each spouse's nonwage income.

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lated with wages and/or nonwage income and hours of work, thus creating spurious correlation between right hand-side variables and the error terms of the hours equations. We thus follow Mroz (1987) and use a second-order polynomial in age and education to instrument the wages, the nonwage income and the number of preschoolers and school age children.[15] Other instruments include father education and dummy variables for race, spanish origin, religion, city size and region of residence. In the unrestricted version, there are 29 parameters to estimate and over 68 instruments (see Tables 2 and 3 for the complete list of instruments). All specifications assume the absence of correlation between the error terms of male and female labor supplies.

The first two columns of Table 2 present the parameter estimates of equations (3.1) and (3.2), respectively, *i.e.* the unrestricted model.[16] Notice that all the parameters are statistically significant at conventional levels, except for nonwage income in women's equation and sex ratio × preschoolers in men's equations.

The parameter estimates of the unrestricted model provide interesting results that are worth mentioning. For instance, the parameter estimate associated with the sex ratio is negative in women's equation and positive in men's equation. Furthermore, the interaction between sex ratio and the presence of preschoolers has a positive impact on women's labor supply, but none on men's labor supply. Put differently, women with preschoolers seem totally unaffected by status of the marriage market. In fact, the null assumption of no impact can not be rejected. On the other hand, women without preschoolers tend to work less, whereas men appear to work more when the sex ratio increases. These results thus reject an important restriction of the unitary model according to which no distribution factor influences behavior.[17] Moreover it also rejects the simple version of the 'separate spheres' model which assumes that the threat point is not divorce but an uncooperative marriage.[18]

[15]The parameter estimates of wages and nonwage income are rather insensitive to the instrumentation of the children variables. Although no formal tests have been conducted yet, it appears the children variables would probably fail an exogeneity test.

[16]The 'extended model' refers to the fact that the distribution factor appears both by itself and in interaction with the presence of preschoolers in the two labor supply functions.

[17]Of course, one could argue that this test is subject to a specification problem since, as shown earlier, the functional form considered for labor supplies imposes unrealistic behavior when all restrctions of the unitary model are assumed.

[18]Theoretically, one could also test restrictions of this model (or of alternative bargaining models) that stem from the particular formulation of the Nash bargaining program. However, these restrictions are likely to be very difficult to derive formally [see McElroy (1990) and

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It can be argued that these tests are biased since the sex ratio is likely to be correlated with unobserved variables. For instance, spatial variations in the sex ratio could be related to labor markets considerations [Grossbard-Shechtman (1993)]. States that concentrate in 'male' sectors will attrack relatively more men through migration. Therefore, they will have high sex ratios and presumably high male hours of work. Female hours of work, on the other hand, may well be below the national average in these states. Conversely, states that concentrate in 'female' sectors will have low sex ratios and high female hours of work.[19] One answer to this criticism is that, as long as these effects influence hours of work through the wage rates, they are taken into account in our model since we control for individual wage rates. However, they could also affect unobserved variables that influence hours of work such as labor market rationing, nonwage remuneration *etc.* One way to discriminate between the marriage market and the labor market hypotheses is to analyse the impact of the sex ratio on the labor supply of singles. According to the marriage market hypothesis, the sex ratio should have no effect on their labor supply (at least if one ignores its impact on transfers to potential spouses). In contrast, under the labor market hypothesis, the sex ratio should influence the labor supply of both singles and couples. Yet, in regressions not reported here, singles (males and females) do not respond to variations in sex ratios. Thus, although the sex ratio may partly reflect conditions on the labor market, it probably is not the whole story.

The restrictions imposed by the collective framework on the generalized semi-log system boil down to the first two equations of the system (3.3), since $f_{7k} = m_{7k} = 0$, for $k = 2, \cdots, K$. To assess the validity of the collective model, we report the parameter estimates of the restricted model in columns (3) and (4) of the table. The bottom row reports the value of both functions. These values follow a χ^2 distribution with as many degrees of freedom as there are overidentifying restrictions. As mentioned earlier, there are 68 instruments used in the estimation and there are 29 parameters to estimate in the unrestricted version of the model. Thus, the critical value is $\chi^2_{0.05}(39) \simeq 54.6$. Consequently, our instruments cannot be rejected on the basis of this test. The restricted estimation also cannot be rejected $[\chi^2_{0.05}(40) \simeq 55.7]$. In the GMM framework, the likelihood ratio test is

Chiappori (1992) for a recent discussion].

[19]Another interpretation or the labor market hypothesis is that in States where one observes a relative scarcity of men (low sex ratio), the demand for their services will be stronger and therefore they will tend to work longer hours than elsewhere. The contrary will be observed for women. Our empirical evidence reject such an interpretation since we observe the opposite results.

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equivalent to the difference in function values, which is distributed as a $\chi^2(k)$, where k is the number of restrictions. Thus, the model is at the margin of being rejected at $\alpha = 5\%$ ($\chi^2_{0.05}(2) = 5.99$). Furthermore, most parameter estimates of the men's equation are not statistically significant.

Column (5) of the table reports the implicit parameters of the sharing rule as derived from the restricted parameters of columns (3) and (4). Since many parameter estimates are not statistically significant we will not discuss the sharing rule in the extended model. Instead, we now turn to Table 3, which reports the parameter estimates of the 'simple model', *i.e.* a specification that omits the interaction term between sex ratio and the presence of preschoolers.

The set up of Table 3 is similar to that of Table 2. Note that nearly all the parameter estimates of the upper panel are statistically significant at conventional levels. It is also worth stressing that the parameter estimates of both tables are relatively similar. As in the previous table, the overidentifying restrictions cannot be rejected on the basis of the usual χ^2 tests. The likelihood ratio test at the bottom of the table indicates that the restriction imposed by the collective model is easily accepted.[20] As in the previous table, the sex ratio has a negative and significant impact on women's hours or work and a positive and significant impact on men's hours of work. Finally, the identifiable parameters of the sharing rule are all statistically significant.

In order to gain insight into the interpretation of the parameters of the sharing rule (see equation (3.4), Table 4 reports the partial derivatives of the sharing along with their standard errors. The first column of the table replicates last column of Table 3. The second column reports the partial derivatives themselves. These partial derivatives represent the impact of a marginal change in one variable on the nonwage income accruing to the husband after sharing. According to our parameter estimates, a one dollar increase in the wife's wage rate, ω_f, translates into less income being transferred to the husband. At the mean of the sample, the impact is relatively small, 85$, and not statistically significant. On the other hand, a one dollar change in the husband's wage rate, ω_h, translates into significantly more nonwage income being transferred to the husband. Indeed, the table shows that as much as 472$ will be transferred from his wife following the increase in ω_h, at the sample mean. However, since the interactive term in the logs of wage rates is negative, this means that an increase in the wife's wage rate reduces the importance of this transfer. These results lend support to the notion that spouses may be involved in some form of bargaining. The next line indicates that a one

[20]In the simple model, the only restriction to satisfy is $\frac{m_4}{f_4} = \frac{m_5}{f_5}$.

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dollar increase in family nonwage income will increase the husband's income by 63 cents. Finally, a one percentage point increase our sex ratio index, according to our parameter estimates, induces men to transfer an additional 277$ (1988) of nonwage income to their spouse.

Finally, the other columns of Table 4 report various labor supply elasticities. In general these elasticities are comparable to those found in the empirical labor supply literature. Men's labor supply elasticity with respect to nonwage income is positive, which is somewhat surprising. Similar results were recently obtained by Smith Conway (1997) also using PSID data. The last two columns report the own wage elasticities for a given level of nonwage income received by each spouse. These elasticities are obtained from individual preferences alone since they ignore the effect of wage rates on the intra-household decision process. Women's elasticity (=0.165) is slightly higher than the one reported in the previous columns. This simply reflects the fact that a marginal increase in their wage rate increases their share of the nonwage income, which in turn reduces labor supply through an income effect. As for men, the wage elasticity has the a negative sign. But since the income effect is positive in their case and sufficiently important, a marginal increase in their wage rate shows up as positive in the previous columns.

6. Conclusion

In this paper, we provide evidence that the marriage market, as proxied by a sex ratio index, influences household labor supplies, specially in the case of household with no pre-school children. Our results thus contradict one important prediction of the unitary model according to which no distribution factor affects intra-household decisions. Rather, we provide some support to bargaining models in which the alternative option to a cooperative marriage is divorce or separation. Moreover, our results do not reject the restrictions imposed by the collective model of labor supply developped by Chiappori (1992), according to which the intra-household outcomes are Pareto-efficient. We also recover most parameters of the sharing rule associated with this model. In particular, according to our results, a one percentage point increase in the percentage of males in the population induces the husbands to increase their transfer to their wife by 277$ (1988) on average.

Our analysis is subject to some limitations though. Since our estimates are conditional to the choice of living in couple, they could suffer from a selection bias. For instance, in regions where the sex ratio is relatively small, more 'low-

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quality' men are likely to marry, given the scarcity of men in the marriage market. However, as long as these men are also characterized by a low productivity in the labor market or low preferences for work, this may create a spurious (positive) correlation between the sex ratio and male hours of work. More research on collective models that endogenize both marital choices and labor supply is clearly needed.

Further, our approach assumes that the sex ratio is exogenous. However it could be viewed as a variable that adjusts across regions to equilibrate the marriage markets [Becker (1991)]. While we discuss some evidence that suggest that this interpretation is not likely to seriously bias our estimates, it would be important to provide an analysis of the factors that explain variations of the sex ratio across regions. Another useful extension of our model would be to introduce a variable of sex ratio *at the beginning of marriage*. As long as marriage agreements are binding, this variable should be the relevant one to explain the intra-household distribution process and labor supply behavior. Finally, introducing household production in the model would be worth considering [see Apps and Rees (1997) and Chiappori (1997) for a discussion].

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APPENDIX : Proof of Proposition 2

A. One distribution factor

Start from :

$$h^1 = H^1\left(w_1, \phi\left(w_1, w_2, y, s, z\right), z\right)$$
$$h^2 = H^2\left(w_2, y - \phi\left(w_1, w_2, y, s, z\right), z\right)$$

Then :

$$A = \frac{h^1_{w_2}}{h^1_y} = \frac{\phi_{w_2}}{\phi_y}$$
$$B = \frac{h^2_{w_1}}{h^2_y} = \frac{-\phi_{w_1}}{1 - \phi_y}$$
$$C = \frac{h^1_s}{h^1_y} = \frac{\phi_s}{\phi_y}$$

and

$$D = \frac{h^2_s}{h^2_y} = \frac{-\phi_s}{1 - \phi_y}$$

Assume that $C \neq D$. Then the last two equations give :

$$\phi_y = \frac{D}{D - C}$$
$$\phi_s = \frac{CD}{D - C}$$

Then the first two lead to :

$$\phi_{w_1} = \frac{BC}{D - C}$$
$$\phi_{w_2} = \frac{AD}{D - C}$$

These partials are compatible if and only if they satisfy the usual cross derivative restrictions. Hence, the following conditions are necessary and sufficient :

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$$\frac{\partial}{\partial s}\left(\frac{D}{D-C}\right)=\frac{\partial}{\partial y}\left(\frac{CD}{D-C}\right)$$

$$\frac{\partial}{\partial w_1}\left(\frac{D}{D-C}\right)=\frac{\partial}{\partial y}\left(\frac{BC}{D-C}\right)$$

$$\frac{\partial}{\partial w_2}\left(\frac{D}{D-C}\right)=\frac{\partial}{\partial y}\left(\frac{AD}{D-C}\right)$$

$$\frac{\partial}{\partial w_1}\left(\frac{CD}{D-C}\right)=\frac{\partial}{\partial s}\left(\frac{BC}{D-C}\right)$$

$$\frac{\partial}{\partial w_2}\left(\frac{CD}{D-C}\right)=\frac{\partial}{\partial s}\left(\frac{AD}{D-C}\right)$$

$$\frac{\partial}{\partial w_2}\left(\frac{BC}{D-C}\right)=\frac{\partial}{\partial w_1}\left(\frac{AD}{D-C}\right)$$

If these equations are fulfilled, then ϕ is defined up to an additive constant; then the knowledge of Marshallian demands allows to recover preferences in the standard way.

B. Several distribution factors

Finally, if there are several distribution factors, then they can enter labor supply functions only through the same function ϕ. This implies that :

$$\frac{h^1_{s_l}}{h^1_{s_1}}=\frac{\phi_{s_l}}{\phi_{s_1}}=\frac{h^2_{s_l}}{h^2_{s_1}}$$

for all l. ■

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TABLE 1
MEANS OF THE DATA
(Standard Deviation in Parentheses)

	Women	Men
Endogenous Variables:		
Hours of Work	1659.9	2241.6
	(650.2)	(629.4)
log-wage	2.01	2.41
	(0.68)	(0.61)
Non-Labour Income	1558.5	
	(6774.9)	
Children ($\leq$ 6)	0.45	
	(0.69)	
Children (7 - 17)	0.88	
	(1.05)	
Exogenous and Instrumental Variables		
Age	35.6	38.0
	(8.4)	(8.9)
Schooling	5.1	5.2
	(1.5)	(1.6)
Father Schooling	3.7	3.6
	(1.8)	(1.7)
Spanish	0.03	0.02
White	0.73	0.73
Black	0.25	0.25
Catholic	0.21	0.21
Jewish	0.02	0.02
Protestant	0.64	0.62
Other Religion	0.05	0.08
Sex Ratio (White)	0.499	
	(0.016)	
Sex Ratio (Black)	0.509	
	(0.082)	
Sex Ratio (Other)	0.477	
	(0.038)	
City $\geq$ 500	0.41	
City 100 - 500	0.25	
City 50 - 100	0.07	
City 25 - 50	0.26	

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TABLE 2

GMM PARAMETER ESTIMATES – EXTENDED MODEL

	Unconstrained Version		Constrained Version		Sharing Rule
	Women	Men	Women	Men	
$\log \omega_f$	1.241	0.418	1.203	-0.107	-0.024
	(4.238)	(1.764)	(3.670)	(0.644)	(1.112)
$\log \omega_h$	0.760	0.383	0.731	0.035	2.979
	(3.082)	(1.667)	(2.687)	(0.181)	(1.953)
$\log \omega_f \times \log \omega_h$	-0.433	-0.180	-0.418	0.014	-1.036
	(4.025)	(1.944)	(3.450)	(0.206)	(1.982)
Non-Labour Income	-0.008	0.024	-0.009	0.023	0.977
	(0.522)	(2.004)	(0.589)	(1.946)	(21.403)
Sex Ratio	-2.545	2.407	-2.536	n.a.†	-0.020
	(3.297)	(3.244)	(2.990)	n.a.	(0.484)
Sex Ratio × Children (≤ 6)	2.392	0.349	2.391	n.a.†	-0.267
	(2.030)	(0.383)	(1.949)	n.a.	(1.071)
Intercept	1.315	-0.080	1.413	2.444	
	(1.538)	(0.101)	(1.412)	(0.594)	
Children (≤ 6)	-1.168	0.006	-1.188	-0.097	
	(2.311)	(0.017)	(2.257)	(0.618)	
Children (7–17)	-0.010	0.063	-0.012	0.022	
	(0.375)	(2.578)	(0.409)	(1.032)	
Education	-0.009	0.020	-0.008	0.024	
	(0.577)	(1.735)	(0.473)	(2.104)	
Age	-0.101	0.012	-0.107	-0.078	
	(2.002)	(0.346)	(1.956)	(3.125)	
North East	-0.125	-0.043	-0.125	-0.041	
	(3.717)	(1.396)	(3.597)	(1.301)	
North Central	-0.033	-0.015	-0.027	0.018	
	(0.818)	(0.508)	(0.661)	(0.641)	
West	-0.062	-0.107	-0.060	-0.059	
	(1.426)	(2.924)	(1.293)	(1.742)	
σ	0.261		0.265		
	(0.518)		(0.620)		
Value of Function	27.781		33.638		
LR Test	5.857				

Notes:

1. *Absolute value of T-statistics in parentheses.*
2. † *This parameter is constrained.*
3. *Instruments: Second order polynomial in age and education (M-F), Father Education (M-F), White (M-F),Spanish (M-F), City size (3 dummies), North-East, North-Central, West, Protestant (M-F), Jewish (M-F),*

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TABLE 3
GMM PARAMETER ESTIMATES – SIMPLE MODEL

	Unconstrained Version		Constrained Version		Sharing Rule
	Women	Men	Women	Men	
$\log \omega_f$	1.379	0.425	1.312	0.469	16.698
	(4.712)	(1.844)	(4.788)	(2.399)	(2.278)
$\log \omega_h$	0.828	0.410	0.780	0.485	13.336
	(3.431)	(1.823)	(3.382)	(3.119)	(1.985)
$\log \omega_f \times \log \omega_h$	-0.474	-0.184	-0.449	-0.208	-5.720
	(4.438)	(2.038)	(4.491)	(2.967)	(2.321)
Non-Labour Income	-0.029	0.022	-0.030	0.023	0.621
	(1.501)	(2.036)	(1.581)	(2.098)	(3.128)
Sex Ratio	-1.913	2.443	-2.119	n.a.†	-26.969
	(2.760)	(3.233)	(3.594)	n.a.	(2.236)
Intercept	0.769	-0.125	1.042	-0.181	
	(0.926)	(0.157)	(1.503)	(0.300)	
Children (≤ 6)	-0.308	0.126	-0.341	0.129	
	(2.107)	(1.092)	(2.482)	(1.463)	
Children (7–17)	-0.003	0.061	-0.007	0.060	
	(0.127)	(2.490)	(0.287)	(2.651)	
Education	-0.005	0.018	-0.004	0.017	
	(0.323)	(1.584)	(0.236)	(1.525)	
Age	-0.096	0.010	-0.106	0.007	
	(2.056)	(0.262)	(2.422)	(0.259)	
North East	-0.130	-0.046	-0.127	-0.048	
	(4.028)	(1.456)	(3.956)	(1.564)	
North Central	-0.019	-0.016	-0.015	-0.019	
	(0.513)	(0.537)	(0.392)	(0.670)	
West	-0.041	-0.109	-0.034	-0.111	
	(0.946)	(2.939)	(0.822)	(3.200)	
σ	0.350		0.342		
	(0.685)		(0.774)		
Value of Function	30.799		31.719		
LR Test		0.920			

Notes:

1. *Absolute value of T-statistics in parentheses.*
2. † *This parameter is constrained.*
3. *Instruments: Second order polynomial in age and education (M-F), Father Education (M-F), White (M-F),Spanish (M-F), City size (3 dummies), North-East, North-Central, West, Protestant (M-F), Jewish (M-F), Catholic (M-F)*

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TABLE 4
SHARING RULE AND ELASTICITIES – SIMPLE MODEL

Variable	SHARING RULE		ELASTICITIES				
			Unconstrained version		Constrained version		Condit on ϕ
	$\phi(\cdot)$	$\frac{\partial\phi}{\partial \text{Variable}}$ †	Women	Men	Women	Men	Women
$\log \omega_f$	16.698	-84.909	0.139	-0.008	0.135	-0.014	0.165
	(2.278)	(1.223)	(2.176)	(0.240)	(2.111)	(0.467)	(1.144)
$\log \omega_h$	13.336	471.820	-0.075	0.016	-0.074	0.028	
	(1.925)	(1.914)	(1.320)	(0.333)	(1.296)	(0.649)	
$\log \omega_f \times \log \omega_h$	-5.720						
	(2.321)						
Non Labor Income	0.621	0.623	-0.027	0.015	-0.027	0.016	
	(2.321)	(3.047)	(1.506)	(2.065)	(1.582)	(2.117)	
Sex-Ratio	-26.969	-277.085					
	(2.236)	(2.136)					

Notes:

1. *Absolute value of T-statistics in parentheses.*
2. † *The derivatives are computed with respect to the ω_f and ω_h, not with respect to $\log \omega_f$ and $\log \omega_h$.*

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ANNEXE 4

Modèle général de demande de bien et d'offre de travail dans un cadre collectif

Efficient Intra-Household Allocations : A General Characterization and Empirical Tests*

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July 1997

Abstract

The neo-classical theory of demand applies to individuals yet in empirical work it is usually taken as valid for households with many members. This paper explores what the theory of individuals implies for households which have more than one member. We make minimal assumptions about how the individual members of the household resolve conflicts. All we assume is that however decisions are made, outcomes are efficient. We refer to this as the collective setting. We show that in the collective setting household demands must satisfy a symmetry and rank condition on the Slutsky matrix. We also present some further results on the effects on demands of variables that do not modify preferences but that do affect how decisions are made.

We apply our theory to a series of surveys of household expenditures from Canada. The tests of the usual symmetry conditions are rejected for two person households but not for one person households. We also show that income pooling is rejected for two person households. We then test for our collective setting conditions on the couples data. None of the collective setting restrictions are rejected. We conclude that the collective setting

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1. Introduction

When considering household behaviour and welfare it is almost universally assumed that the many person household can be treated as though it has a single set of goals. The adoption of this 'unitary' model is very convenient, if only because standard tools of consumer analysis can then be applied at the household level. Methodologically, however, it stands on weak grounds. Neo-classical utility theory applies to individuals and not to households[1]. There is also mounting empirical evidence that the unitary model does not hold. In particular the fundamental observable implication of utility theory - symmetry of the Slutsky matrix - is regularly rejected on household data (see, for example, Blundell, Pashardes and Weber (1993) and Browning and Meghir (1991)). Further disquiet is given by the universal rejection of the 'income pooling' property of the unitary model; that is, the implication that the source of household income should not have any effect on allocations once we condition on total expenditure (see, for instance, Thomas (1990), Schultz (1990), Bourguignon, Browning, Chiappori and Lechene (1993), Phipps and Burton (1992) and Lundberg, Pollak and Wales (1995)).These rejections have either been seen as a rejection of utility theory or have been attributed to specification problems (inadequate functional forms, inappropriate separability assumptions, misspecification of the stochastic structure and so on). Thus it has been concluded either that utility theory is false or that it is untestable.

Our answer to these 'problems' with neo-classical utility theory is completely different. We claim that the theory has not been taken seriously enough. We start from the premise that utility theory does apply, but only to individuals and not to households. In this paper we present a general characterization of an alternative model of household behaviour to the unitary model, namely the 'collective' model suggested in Chiappori (1988a and 1992). The two assumptions of the collective model are that each person in the household has his or her own preferences and that collective decisions are Pareto efficient. Under these assumptions, we exhibit

[1]Two major contributions have tried to reconcile the unitary model with the fact that households may consist of more than one decision maker. However, Samuelson's (1956) idea of a household welfare function relies upon the *ad-hoc* idea that the latter (and in particular the respective weights given to each member's utility) is independent of prices and incomes. While Becker's 'rotten kid' theorem (see Bergstrom (1989) for a statement) appears sounder, it still requires transferable preferences and a specific decision process to hold true.

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a set of theoretical properties that have to be fulfilled by household demands, and can thus be seen as a generalization to the multi-person setting of Slutsky symmetry in the unitary framework. We then test the conditions on a sample of Canadian households.

The idea of explicitly modelling household behaviour as a collective process can be traced back at least to Becker's seminal work (see Becker (1991) for a recent exposition). Also, it has been clear for some time that a multi-person approach might well (and actually should) lead to violations of the predictions from the unitary model. For instance, this point is emphasized by Bourguignon (1984) within a non-cooperative setting and Pollak (1985) using a 'transaction approach'.

Several models have explicitly modelled intra-household decision making as a cooperative process. The Nash-bargaining representation of family decisions, as initiated by Manser and Brown (1980) and McElroy and Horney (1981), is of particular interest for our present purpose. At the core of this approach are two interesting ideas. One is that, within a collective framework, household demands should be sensitive to the intra-household distribution of resources, and more generally to any environmental variable that may influence the decision process - say, through a shift in threat points ('EEP's' in McElroy (1990) terminology, or 'distribution factors' in Browning *et al* (1994)). This has given rise to the literature on testing for 'income pooling' that was referenced above. In previous papers with other co-authors (Bourguignon *et al* (1993), Browning, Bourguignon, Chiappori and Lechene (1994) and Bourguignon, Browning and Chiappori (1995)) we contributed to this line of research by investigating what could be learned from conventional family expenditure data about what goes on inside the household. In the third paper, in particular, we showed that the collective setting imposes testable restrictions upon the way in which distribution factors can enter demand equations; moreover, we investigate the conditions under which the observation of household demands enables us to identify individual Engel curves and the form of the decision process. The key point, however, is that this analysis requires only cross-section variation in the data; that is, we did not exploit any price variation.

More relevant for the present paper is the second intuition put forth in the Nash-bargaining literature - namely, that the repeated rejections of Slutsky symmetry in empirical work may occur because household decisions cannot be crammed into an overly restrictive unitary framework. This suggest that the case where price variations can be observed deserves careful investigation. In this framework, a very natural question arises: can one derive restrictive, testable implications

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of the Nash-bargaining framework upon demand functions, that could be seen as the counterpart (or, more precisely, the generalization) of Slutsky symmetry and negativeness in the unitary case? This is precisely the topic of the present paper.

Important as it is, it is fair to say that this question had not receive a convincing answer so far (see Chiappori (1988b, 1991) and McElroy and Horney (1990)). One contribution of the present paper is to fill this gap. In what follows, we actually solve a more general problem - namely, what does the efficiency assumption alone imply for household demands, and specifically for the form of the Slutsky matrix?

Though we do not formally justify the efficiency assumption, we do believe that it has a good deal of intuitive appeal. For one thing, the household is one of the preeminent examples of a repeated 'game' so that we feel justified in assuming that each person knows the preferences of the other people in the household. Given this symmetry of information and the fact that the game is repeated it is plausible that agents find mechanisms to support efficient outcomes; as it is well-known, cooperation often emerges as a long-term equilibrium of repeated non-cooperative frameworks[2]. A second point is that efficiency is probably the most natural generalization to the multi-person setting of utility maximization in standard models. In particular, the collective model we consider includes the unitary representation as a (very) special case; hence, the conditions we derive generalize in a straightforward way Slutsky symmetry - a fact that leads directly to nested tests. Finally, axiomatic models of bargaining with symmetric information generally assume efficient outcomes. This is the case, for instance, of all models developed so far in the Nash-bargaining approach. In other words, the 'collective' framework we consider in this paper encompasses all cooperative models existing in this literature. As a consequence, the conditions we shall derive from the efficiency assumption alone apply, *a fortiori*, to all these models as well[3].

Our main purpose is the derivation of testable implications of the collective framework. An immediate implication is that we must adopt the least restrictive

[2]This is not to say, however, that we cannot envision circumstances that would lead to inefficient outcomes. Clearly, if there is asymmetric information (for example, one partner can consume some goods without the other partner knowing) then the case for efficiency is weakened. In the end this is an empirical matter: what does the collective setting imply for household behaviour and are these predictions rejected by the data? This paper is directed to these issues.

[3]The specific concept of Nash-bargaining can actually be viewed as a way of determining the location of the final outcome in the Pareto set. Whether this particular assumption implies additional restrictions upon observed behavior is still an open question

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set of assumptions possible. Ideally, the conditions we are looking for should result from the efficiency axiom only, with no additional ('auxiliary') assumption required. In this spirit, we do not suppose that the econometrician can determine which goods are private and which public within the household; any commodity may be either public, or private, or both. Moreover, we do not assume that the individual consumption of private goods is observable. Similarly, we do not introduce any particular assumption on individual preferences, except that they can be represented by conventional utility functions. That is, we allow for intra-household consumption externalities, altruism, etc..

Despite this explicitly minimalist set of assumptions, we show that one can make very specific predictions about household behaviour. The principal theoretical result of the paper is that although Slutsky symmetry need not hold in the collective setting, it can be generalized in a straightforward way; namely, the Slutsky matrix has to be equal to the sum of a symmetric matrix and a rank one matrix. This strong theoretical property is a consequence of the efficiency hypothesis alone.

This basic result is presented in Section 2. In section 3 we extend the analysis in three different directions. The most important of these extensions is to allow for distribution factors (as alluded to above), formally defined as variables which do not enter individual utilities directly but that do affect distribution within the household. It turns out that the collective model implies that there is a close relationship between the influence of such variables on demand and price responses.

The second part of the paper is empirical, and is aimed at testing our predictions on household data. From a general viewpoint, the case of price variations that we consider here has implications for two areas: demand analysis on time series of family expenditure surveys (for example, the U.K. FES or the U.S. CEX) and the analysis of labour supply on cross-sections (or panel data) where the prices that vary across individuals are wages. Although the latter is the more important application, we have chosen initially to concentrate on the former since the analysis of labour supply for individuals raises many problems that are less pressing in the demand case (for example, wages may be non-linear, endogenous and unobserved for some individuals).

In section 4 we present a flexible parametric demand system and derive the implications of the predictions of the previous sections for the parameters of this system. This includes a novel analysis of testing for the rank of a matrix in our context. In section 5 we present empirical results using the Canadian Family

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Expenditure Survey (FAMEX) data on single person households and households containing just a married couple. We first show that Slutsky symmetry is not rejected for singles but it is for couples. To the best of our knowledge this is the first time that anyone has shown that symmetry is not rejected for singles. We then go on to test the predictions of the collective setting derived in sections 2 and 3 on the couples data. We do not reject any of these restrictions. This provides strong, though preliminary, support for our view that the collective model is a viable alternative to the unitary model. In the concluding section we discuss some possible areas of future research.

2. Theory - The general case

2.1. The collective setting

2.1.1. Preferences

We consider a two person (A and B) household. Household purchases[4] are denoted by the n-vector $\mathbf{q}$ with associated market price vector $\mathbf{p}$. Household demands are divided between three uses : private consumption by each person, $\mathbf{q}^A$ and $\mathbf{q}^B$, and public consumption $\mathbf{Q}$. Each good may serve several uses simultaneously[5]; public and private consumption vectors are only linked by :

$$\mathbf{q}^A + \mathbf{q}^B + \mathbf{Q} = \mathbf{q} \tag{2.1}$$

The household budget constraint is :

$$\mathbf{p}'(\mathbf{q}^A + \mathbf{q}^B + \mathbf{Q}) = \mathbf{p}'\mathbf{q} = x \tag{2.2}$$

where x denotes total expenditure.

As said before, we adopt a Beckerian framework in which each member has her or his own preferences over the goods consumed in the household. Whether consumption of a particular good by a particular person is, by nature, private, public or both is irrelevant for our results. Also, each member's preferences can

[4]Formally purchases could include leisure (so that the price vector includes the wages - or virtual wages for non-participants - of A and B). As already indicated, we shall not be emphasizing the implications of our results for labour supply. Also, we only consider a static model, and assume that all goods are non-durables.

[5]For instance, expenditures on 'telephone services' includes a public element (the rental) and a private element (the actual use of telephone).

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depend on both members' private and public consumption (the 'altruistic' case in Bourguignon and Chiappori (1992)); this allows for altruism, but also for externalities or any other preference interaction. Our results are consistent with all possible interactions. We only assume that preferences, defined on $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$, are 'well-behaved' in the usual sense :

Axiom 1. *Member I's preferences ($I = A, B$) can be represented by a utility function of the form $u^I(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$ that is strongly concave and twice differentiable in $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$, and strictly increasing in $(\mathbf{q}^I, \mathbf{Q})$.*

Note that we do not impose that $u^I(.)$ is increasing in $\mathbf{q}^J$ for J not equal to I; that is, we allow for selfishness or even negative consumption externalities between members.

2.1.2. The decision process

We now consider the mechanism that the household uses to decide on what to buy. Note, first, that if the functions u^A and u^B represent the same preferences then we are back in the conventional 'unitary' model; then the common utility is maximized under the budget constraint. Alternatively, we could assume that one of the partners can impose her (or his) preferences and use the corresponding utility function in the traditional way; this also yields a unitary model. But these are highly specific assumptions. In general, the 'process' that takes place within the household is more complex.

As stated in introduction, our approach at this point is axiomatic; we postulate efficiency, as expressed in the following axiom :

Axiom 2. *The outcome of the household decision process is Pareto efficient; that is, for any price-income bundle $(\mathbf{p},x)$, the consumption vector $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$ chosen by the household is such that no other vector $(\bar{\mathbf{q}}^A, \bar{\mathbf{q}}^B, \bar{\mathbf{Q}})$ in the budget set could make both members better off.*

Following Chiappori (1992), we refer to models that allow for different preferences with efficiency as the 'collective' setting. Finally, we add some structure by assuming the following:

Axiom 3. *There exists a differentiable, zero-homogeneous function $\mu(\mathbf{p},x)$ such that, for any $(\mathbf{p},x)$, the vectors $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$ are solutions to the program :*

$$\max_{\mathbf{q}^A,\mathbf{q}^B,\mathbf{Q}} \mu(\mathbf{p},x).u^A(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}) + [1 - \mu(\mathbf{p},x)] .u^B(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}) \qquad (2.3)$$

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$$\mathbf{p}.(\mathbf{q}^A + \mathbf{q}^B + \mathbf{Q}) = x$$

As it is well-known, any point on the Pareto frontier can be obtained as a solution to a program of this type (for some well-chosen μ). Axiom 3 essentially postulates that the decision process always has a ***unique, well-defined outcome***; or, in other terms, that there exists a ***demand function*** (and that, in addition, the latter is continuous and zero-homogeneous). Homogeneity is uncontroversial; it essentially means that expressing prices and incomes in cents instead of dollars does not change actual behaviour. The smoothness assumption is standard, and made for analytical convenience[6]. Uniqueness, on the other hand, is a real assumption, albeit not an extremely restrictive one. Two points should in particular be emphasized :

- the assumption is consistent with our general framework, which postulates efficiency. Indeed, a natural (although not exclusive) justification is that the members play some *cooperative* game under symmetric information. In most cases, this should lead to a unique outcome[7]. Note, in particular, that all bargaining models developed in the literature exhibit the same property, since they are based upon a specific bargaining equilibrium concept (Nash, Kalai-Smorodinsky,...).
- from an applied viewpoint, assuming the existence of a demand function does not seem unduly restrictive. To the best of our knowledge, most (if not all) existing empirical work on demand relies upon a similar assumption.

The 'distribution' function μ summarizes the decision process. Take some given utility functions u^A and u^B. Then the budget constraint defines, for any price-income bundle, a Pareto frontier. From Axiom 2, the final outcome will be located on this frontier. Then μ determines the final location of the demand vector on this frontier.

[6]The key point that drives the results is that the set of efficient outcomes is one-dimensional - a property that is independent of any smoothness (or even uniqueness) assumption. However, its more natural (and more tractable) development is in terms of $(n-1)$ dimensional manifolds, which requires smoothness.

[7]This is sharp contrast with non cooperative games, or with models of bargaining under asymetric information - where multiplicity of equilibria is more difficult to rule out. Of course, such models are in general incompatible not only with the uniqueness assumption, but with Axiom 2 as well, since the outcome will typically violate efficiency.

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The parameter μ has an obvious interpretation as a 'distribution of power' function. If $\mu = 1$ then the household behaves as though A always get their way, whereas if $\mu = 0$ it is as though B is the effective dictator. For intermediate values, the household behaves as though each person has some decision power. Note that μ will generally depend on prices and total expenditures, since these environmental variables influence the distribution of 'power' within the household.

Two additional points may be noted at this stage. One is that, in general, μ may also depend on other factors, such as the individual incomes of the two partners, or any factor of the household environment that may affect the decision process ('distribution factors' in Browning *et al* (1994)). This idea is explored in the next section; for the moment, let us first investigate the properties of the basic model. Also, assume preferences are identical. Then we are back in the unitary setting and μ is not defined. However, we can then use the convention that $\mu = 0$ (or, as a matter of fact, any other convention).

Any given (demand) function $\mathbf{q}(\mathbf{p},x)$ is said to be *compatible with collective rationality* if and only if there exist functions $\mathbf{q}^A(\mathbf{p},x)$, $\mathbf{q}^B(\mathbf{p},x)$, $\mathbf{Q}(\mathbf{p},x)$, solution of a program of the type (2.3), such that $\mathbf{q}(\mathbf{p},x) = \mathbf{q}^A(\mathbf{p},x) + \mathbf{q}^B(\mathbf{p},x) + \mathbf{Q}(\mathbf{p},x)$. A first property of such functions is given by the following result :

In the following, our goal is to derive additional properties of these functions.

2.1.3. Household utility

The next step is to define what we shall call the household utility function. The latter will be reminiscent of the unitary setting, but with the difference that it will depend on μ. Formally :

Definition 1. *In the collective setting, the household utility function is defined as :*

$$u^H(\mathbf{q},\mu) = \max_{\mathbf{q}^A,\mathbf{q}^B,\mathbf{Q}} \mu.u^A(\mathbf{q}^A,\mathbf{q}^B,\mathbf{Q}) + (1-\mu).u^B(\mathbf{q}^A,\mathbf{q}^B,\mathbf{Q}) \qquad (2.4)$$

subject to

$$\mathbf{q}^A + \mathbf{q}^B + \mathbf{Q} = \mathbf{q}$$

Clearly, the maximization of the household utility under the budget constraint will lead to the same demand function as program (2.3) above. Two points must be stressed here :

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- The household utility function u^H will depend on prices and income as soon as μ is a function of these variables. So we are in a case of price-dependant preferences, which explains why the usual results of consumer theory (Slutsky symmetry etc.) will no longer hold true in the collective context.

- However, *prices and income enter only through the scalar function* μ. The same will also be true of any other variable that affects the decision process but not preferences. This remark will be crucial in the derivation of the results below.

2.2. Dual representations of the collective program

Given utility functions for the two people we can define a dual representation of 'household' preferences. This can be done in two equivalent ways. First, for any μ, define the household indirect utility function $V(\mathbf{p}, x, \mu)$ as the maximand of the initial optimization problem above :

$$V(\mathbf{p}, x, \mu) = \max_{\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}} \mu u^A(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}) + (1-\mu) u^B(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}) \qquad (2.5)$$

$$\mathbf{p} . (\mathbf{q}^A + \mathbf{q}^B + \mathbf{Q}) = x$$

which can also be written as :

$$V(\mathbf{p}, x, \mu) = \max_{\mathbf{q}} u^H(\mathbf{q}, \mu) \qquad (2.6)$$

$$\mathbf{p}'\mathbf{q} = x$$

In what follows, let $\mathbf{q} = \mathbf{f}(\mathbf{p}, x, \mu)$ denote the solution of this program - that is, the collective counterpart of Marshallian demands; note that $\mathbf{f}(.)$ is a function of μ as well.
Now, we know, from the envelope theorem, that:

$$\frac{\partial V(\mathbf{p}, u, \mu)/\partial p_i}{\partial V(\mathbf{p}, u, \mu)/\partial x} = -f_i$$

which is the equivalent, in the collective setting, of Roy's identity in the unitary case. This means that, *for any constant* μ, an infinitesimal change in one price,

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say dp_i, can be 'compensated' (in the sense that the household utility will not change) by a change in income exactly equal to $dx = q_i.dp_i$. Of course, each member's utility will, in general, change.

The corresponding expenditure function will be defined as :

$$E(\mathbf{p}, u, \mu) = \min_{\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}} \mathbf{p}'(\mathbf{q}^A + \mathbf{q}^B + \mathbf{Q}) \tag{2.7}$$

$$\mu . u^A(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}) + (1 - \mu) . u^B(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q}) \geq u$$

The analogy with traditional duality theory can in fact be pushed somewhat further. First, the expenditure function $E(.)$ is linear homogeneous and concave in $\mathbf{p}$. Also, let $\mathbf{h}(\mathbf{p}, u, \mu)$ denote the solution of program (2.7). Note that $\mathbf{h}(\mathbf{p}, u, \mu)$ can be interpreted as a compensated demand function (since it is the demand that obtains holding household utility constant). It is important to stress, however, that $\mathbf{h}(.)$ is defined as a function of the 'distribution of power' index $\mu(.)$ - that is, μ must also be kept constant.

Again from the envelope theorem, we have:

$$\frac{\partial E(\mathbf{p}, u, \mu)}{\partial p_i} = h_i$$

Duality between programs implies that :

$$\mathbf{f}(\mathbf{p}, E(\mathbf{p}, u, \mu), \mu) = \mathbf{h}(\mathbf{p}, u, \mu)$$

It follows that :

$$\frac{\partial f_i}{\partial p_j} + \frac{\partial f_i}{\partial x} . f_j = \frac{\partial h_i}{\partial p_j}$$

This is equivalent to Slutsky conditions in the unitary case. In particular, the matrix Σ with general term

$$\sigma_{ij} = \left(\frac{\partial f_i}{\partial p_j} + \frac{\partial f_i}{\partial x} . q_j\right)$$

can be interpreted as the partial derivatives of demands with respect to prices, *holding both household utility and the 'distribution of power' index μ constant.*

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2.3. Restrictions on demands

We now derive our main result, which characterizes the properties of observed demand functions. What has to be emphasized here is that *we never observe the function* $\mathbf{f}(\mathbf{p}, x, \mu)$. Indeed, by definition, $\mathbf{f}(\mathbf{p}, x, \mu)$ describes how demands respond to independent variations of $\mathbf{p}, x$ *and* μ. But we do not observe such independent variation. For any given price-income bundle $(\mathbf{p}, x)$, the behaviour we observe corresponds to *one* specific value of μ - namely, the value $\mu(\mathbf{p}, x)$ taken at this point by the specific distribution function that characterizes the household at stake. In other words, what we actually observe is the demand function ξ defined by :

$$\boldsymbol{\xi}(\mathbf{p}, x) = \mathbf{f}\,(\mathbf{p}, x, \mu(\mathbf{p}, x))$$

The question, now, is : which predictions does the collective setting imply for observed demand functions $\boldsymbol{\xi}(\mathbf{p}, x)$? A first, elementary property was given in Proposition **??** above : demands $\boldsymbol{\xi}(\mathbf{p}, x)$ are zero-homogeneous and continuously differentiable in $(\mathbf{p}, x)$ and satisfy adding-up:

$$\mathbf{p}'\boldsymbol{\xi}(\mathbf{p}, x) = x$$

Of course, we are interested in deeper and more structural properties. To derive these, we first define the pseudo-Slutsky matrix associated with $\boldsymbol{\xi}(\mathbf{p}, x)$ as:

$$S = \boldsymbol{\xi}_{\mathbf{p}} + \boldsymbol{\xi}_x \boldsymbol{\xi}'$$

where $\boldsymbol{\xi}_{\mathbf{p}}$ is the $(n \times n)$ Jacobian matrix of partials of $\boldsymbol{\xi}$ with respect to $\mathbf{p}$, and $\boldsymbol{\xi}_x$ the vector of partials of $\boldsymbol{\xi}$ with respect to x. In the unitary setting, S would be symmetric and negative semi-definite. In the collective model, this property generalizes as follows:

$$$

The interpretation of this formula goes as follows. Assume that the price of good j is changed by an infinitesimal amount dp_j, the change being compensated by an increase in income $dx = q_j.dp_j$. What will be the effect of this on the demand for good i ? The formula says that this effect can be decomposed into two components. One corresponds to a substitution effect : holding both household utility V and power index μ constant, the change in price will induce a reallocation of consumption, as defined by the corresponding term in matrix Σ. But, on the top of this, such a change will also modify μ; precisely,

$$d\mu = \frac{\partial \mu}{\partial p_j}.dp_j + \frac{\partial \mu}{\partial x}.dx = (\frac{\partial \mu}{\partial p_j}. + q_j \frac{\partial \mu}{\partial x}).dp_j$$

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- hence the **v** vector. This, in turn, will change consumption of good i by an amount:

$$dq_i = \frac{\partial f}{\partial \mu}.d\mu$$

as indicated by the **u** vector.

A consequence that will be useful in the following is :

Corollary 1. ('SR1 property') *In the collective setting, the pseudo-Slutsky matrix S is the sum of a symmetric, negative semi-definite matrix Σ and a matrix R that has at most rank one.*

This SR1 ('symmetric plus rank one') condition obviously generalizes the unitary model (since $R = 0$ in the latter). This property is somewhat reminiscent of the Diewert-Mantel aggregation restrictions for economies with more goods than agents; see Shafer and Sonnenschein (1982) for an overview.

A geometric interpretation of SR1 is the following. Remember, first, that for any given pair of utilities, the budget constraint defines the Pareto frontier as a function of the price-income bundle; then μ determines the location of the final outcome on the frontier. Assume, now, that prices and income are changed. This has two consequences. For one thing, the Pareto frontier will move. Keeping μ constant, this would change demand, in a way described by the Σ matrix. Note, however, that this change will *not* violate Slutsky symmetry; that is, its nature is not different from the traditional, unitary effect. The second effect is that μ will also change; this will introduce an additional move of demand *along* the (new) frontier. This change (as summarized by the R matrix) *does* violate Slutsky symmetry. But moves along a one-dimensional manifold are quite restricted. For instance, the set of price-income bundles that lead to the *same* μ is likely to be quite large in general; indeed, under our smoothness assumption, it is a $(n-1)$-dimensional manifold. Considering the linear tangent spaces, this means that there is a whole hyperplane such that, if the (infinitesimal) change in prices and income belongs to that hyperplane, then no deviation from Slutsky symmetry can be observed. In other words, *the SR1 condition is a direct consequence of the fact that, in a 2-person household, the Pareto frontier is of dimension 1, whatever the number of commodities.*

2.4. Testing for SR1

How can a property like SR1 be tested ? The result we exploit is that a matrix S is SR1 if and only if the antisymmetric matrix $M = S - S'$ is of rank at most

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2 (remember that a matrix M is antisymmetric if $M' = -M$). A more precise statement is the following :

Lemma 1. • *Let S be some SR1 matrix:*

$$S = \Sigma + \mathbf{u}\mathbf{v}'$$

and assume that S is not symmetric. Then vectors $\mathbf{u}$ and $\mathbf{v}$ are linearly independent; the matrix $M = S - S'$ is of rank 2, and $\mathrm{Im}(M)$ (the subspace spanned by the columns of M) is spanned by $\mathbf{u}$ and $\mathbf{v}$.

- *Conversely, let M be an antisymmetric matrix of rank 2, and let $\bar{\mathbf{u}}$ and $\bar{\mathbf{v}}$ be arbitrary independent vectors of $\mathrm{Im}(M)$. There exists a scalar $\lambda \neq 0$ such that $M = \lambda(\bar{\mathbf{u}}\bar{\mathbf{v}}' - \bar{\mathbf{v}}\bar{\mathbf{u}}') = \mathbf{u}\mathbf{v}' - \mathbf{v}\mathbf{u}'$ where $\mathbf{u} = \lambda\bar{\mathbf{u}}$, $\mathbf{v} = \bar{\mathbf{v}}$. In particular, for any symmetric matrix Σ, the matrix $S = \Sigma + \mathbf{u}\mathbf{v}'$ is such that $M = S - S'$*

What is important for our purpose is that, according to this result, testing for the collective model amounts to testing for the rank of matrix $M = (S - S')$. The collective model (with two decision makers) predicts this rank should be at most two, while it would be zero in the unitary case. This will be crucial in the empirical sections below.

A final remark is that antisymmetry has specific implications for the rank of M. These are given by the following Lemmas:

Lemma 2. *All the eigenvalues of an antisymmetric matrix are imaginary. In particular, an antisymmetric matrix has even rank.*

Lemma 3. *Let $M = (m_{ik})$ be any non zero, antisymmetric matrix, and assume, without loss of generality, that m_{12} is not equal to 0. Then M has rank 2 if and only if, for all (i, k) such that $k > i > 2$,*

$$m_{ik} = \frac{m_{1i}m_{2k} - m_{1k}m_{2i}}{m_{12}}$$

Thus the elements of rows 3 to n in M are functions of the elements of the first two rows (the same is true for columns). Since this characterization only involves parametric restrictions of the familiar sort it is easy to test. Note that for an $(n \times n)$ matrix this involves $(n-2)(n-3)/2$ restrictions. As a benchmark, testing for Slutsky symmetry involves $n(n-1)/2$ restrictions. So, though Slutsky

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symmetry is of course more restrictive, the number of restrictions is of the same order when n is large.

Our findings can be summarized in the following proposition, that underlies the empirical analysis of the next sections :

Proposition 1. *Let S denote the pseudo-Slutsky matrix, and let $M = S - S'$. Then, in the collective setting :*

- *M has rank zero or two,*
- *if M has rank zero, the unitary case cannot be ruled out,*
- *if M has rank 2, then $M = \mathbf{u}\mathbf{v}' - \mathbf{v}\mathbf{u}'$ for two vectors $\mathbf{u}$ and $\mathbf{v}$ that span $\mathrm{Im}(M)$. Moreover, for any vector $\mathbf{w}$ orthogonal to $\mathrm{Im}(M)$ (that is, such that $\mathbf{w}'\mathbf{v} = \mathbf{w}'\mathbf{u} = 0$), then $\mathbf{w}'S\mathbf{w} \leq 0$.*

Note that these tests have a nested structure. Namely, one can first test whether the rank of M is more than two, which would reject the collective model altogether. If it is not rejected, then one can test whether the rank is zero, which would correspond to the unitary model. The collective model predicts that the rank should be zero for singles, but may be two for couples.

2.5. How many goods are needed ?

We have just proved that a given household demand function cannot be compatible with the collective model unless it satisfies the SR1 condition - that is, unless its Slutsky matrix S is such that $M = S - S'$ is of rank at most two. Suppose we observe the household demand for n commodities; what is the minimum value of n for which this property does in fact imply testable restrictions upon demand functions ? In other words, how many commodities do we need to test the SR1 property ?

The answer is given by the following Proposition :

Proposition 2. *Take any n functions $f^i(\mathbf{p}, x)$, $i = 1, ..., n$.*

- *If $n \leq 3$ then the corresponding Slutsky matrix S always satisfies SR1.*
- *If $n \leq 4$ and if the f^i's are zero homogenous in $(\mathbf{p}, x)$ and satisfy adding-up then the corresponding Slutsky matrix S always satisfies SR1.*

The conclusion is that, given the homogeneity assumption above, *one needs at least 5 commodities to test the SR1 property.* This has important implications for modelling labour supply; we discuss this further in the conclusion.

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3. THEORY - EXTENSIONS

In this section we present three extensions to the basic theory of the last section. The first of these extends the theory to households with more than two members. The second extension allows for distribution factors; that is, variables that affect the distribution function μ but not preferences directly. The final extension puts some restrictions on the way prices enter μ.

3.1. Many person households.

If there are more then two people in the household then the class of demands admitted in the collective setting will generally be wider. The exact conditions are given in the next Proposition (the proof follows that of Proposition 2):

Proposition 3. *Assume that the household has $k+1$ members where $k < (n-1)$. In the collective setting the Pseudo-Slutsky matrix S is the sum of a symmetric matrix and a matrix of rank no greater than k (SRk).*

Fairly obviously all of the previous analysis goes through with $(\mu_1 ... \mu_k)$ replacing μ everywhere. This rank condition includes the unitary case and also the two person collective setting.

One possible field of application is to households with children present. To illustrate, suppose the child is named C and let $u^C(.)$ be her utility function. Formally, we can test whether the household behaves as a one, two or three person decision unit by testing for symmetry, SR1 and SR2 respectively. If we reject symmetry but not SR1 then it is as though the household is composed of two decision makers. One obvious choice would be mother and father; this is not to say, of course, that neither parent cares about the child but simply that the child does not have a direct influence on the decision making process. They may, however, have an indirect effect since each parents' preferences over $\left(\mathbf{q}^A, \mathbf{q}^B, \mathbf{q}^C, \mathbf{Q}\right)$ may take into account the child's preferences. Other interpretations are also possible: for example, mother and daughter have the same preferences and father differs.

Identifying intra-household interactions requires more structure than we have so far imposed (see Bourguignon, Browning and Chiappori (1995) for a discussion in the cross-section case) but even the possibility of determining the effective number of decision makers in a household leads to interesting issues. For example, in the adult equivalence scale literature, statements are often made about the amount of income needed to make one household as well off as another. Since it

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is people and not households that have welfare this equating of household welfare is sometimes somewhat murky (but not in all formulations; see, as an exemplary counter-example, Blackorby and Donaldson (1993)). Within the collective framework we can, of course, define household welfare as being the weighted sums of particular utilities. Whether or not we actually want to make this identification between weights that rationalise demands and weights in a social (family) welfare index is another matter. Knowing that father acts as a dictator and discounts the welfare of mother and daughter may not lead us to do the same.

In the multi-person household above we restricted the number of members to be at least two less than the number of goods. The necessary condition in Proposition 5 is no longer restrictive for $k \geq n-1$, since any $n \times n$ matrix can be written as the sum of a symmetric matrix and a matrix of rank $(n-1)$. Though the condition in proposition 5 is only necessary, it is indeed the case that if we have as many people as goods minus one then the collective setting does not impose any restrictions on demand, as stated in the following result:

Proposition 4. *(Chiappori (1990)). Assume that the household has at least* $(n-1)$ *members. For any finite set of prices and demands, one can find preferences for which observed behaviour is compatible with the collective setting.*

The proof relies on known results on aggregate demands for private goods.

3.2. Including distribution factors.

The next extension to the basic model that we discuss in this section is the inclusion of variables that affect the distribution function $\mu(\mathbf{p}, x)$. The obvious examples here are the incomes of the two partners but these variables could also include a host of Extra-Environmental Parameters (EEP's) to use the terminology of McElroy (1990). For example, it might be that changes in divorce law or discrimination against women in the work place have an impact on intra-household decision making (as they shift power within the household). In defining such variables it is most important to identify variables that may affect the μ function but that do not affect preferences directly (that is, that do not enter each person's utility function). We term such variables *distribution factors.* We distinguish such variables from *preference factors* which are variables that affect preferences directly[8].

[8]For convenience we assume that there is no overlap between preference and distribution factors. Thus all variables that affect demands (other than prices and total expenditure) are partitioned between these two groups.

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To take an example, suppose that it is the case that there are fixed costs of going to work that are independent of the wage. Then participation in the labour force could be considered a preference factor and earnings would be a candidate for a distribution factor since demand should not depend on earnings, once we condition on total expenditure and labour force participation. Of course, if the costs of going to work do depend on the wage (for example, high wage jobs require more expensive clothing or higher travel costs) then we cannot take earnings as a distribution factor.

We begin with the case of a single distribution factor y, so that $\mu = \mu(\mathbf{p}, x, y)$. As already discussed this means that y only enters the household utility function through the same index as prices and total expenditure: $u = u(\mathbf{q}, \mu(\mathbf{p}, x, y))$. Household demands take the form $\xi(\mathbf{q}, \mu(\mathbf{p}, x, y))$. Denoting the gradient of demands to changes in y by ξ_y, we have the following conditions on the way this factor can affect demands:

Proposition 5. *Distribution factor linearity*. *In the collective setting, we have the following equivalent conditions:*

1. *the Pseudo-Slutsky matrix takes the form $S = \Gamma + \xi_y \mathbf{v}'$ where Γ is symmetric.*
2. *ξ_y can be written as a linear combination of the columns of $(S - S')$.*

Since S and the vector $\boldsymbol{\xi}_y$ are observable we can use condition (ii) to test for this restriction. Of course, we can only test for condition (ii) conditional on imposing **SR1** on S; without this $(S - S')$ can have full rank and condition (ii) would be satisfied trivially.

Proposition 5 is an unusual result since it relates the response to a change in the distribution factor to price effects 'purged' of the usual Slutsky symmetry. Outside the collective setting there is no particular reason why responses to, say, changes in the relative earnings of the two partners should be related to price responses. Thus this proposition offers a potentially powerful test of the collective setting.

Proposition 5 also has an interesting converse. Suppose that we have some variable y that we are sure would affect demands if the collective model holds but the unitary model does not hold. If we find that this variable does not affect demands (that is, $\boldsymbol{\xi}_y = \mathbf{0}$) then we cannot reject the unitary model. To illustrate, if there is no effect of relative incomes on demand then it must be that households behave as though they are maximising a single utility function

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(since Γ is symmetric). Of course, this test relies on our maintaining that if anything is going to affect intra-household allocation but not preferences then it is relative incomes; if we do not maintain this then this is not a test of the unitary model (that is, $\boldsymbol{\xi}_{\mathbf{y}}$ being zero is only necessary for the unitary model, it is not sufficient). This parallels the tests of the unitary model which test for 'income pooling' (that is, the absence of any effect of incomes on allocation) that have now been performed by many people (see, for example, Thomas (1990), Bourguignon *et al* (1993), Phipps and Burton (1992) and Lundberg *et al* (1995)).

If we do not observe price variation then the presence of a single distribution factor does not impose any restrictions on demands (strictly, Engel curves). Intuitively, this can be seen by noting that the condition in proposition 5 (ii) requires an estimate of S which is only identified if we have price variation. Thus proposition 5 adds to the conditions that are present if we observe price variation. If we add more distribution factors so that **y** is now a vector then the collective setting imposes further restrictions. In Bourguignon *et al* (1995) the following is proved:

Proposition 6. *Distribution factor proportionality*. *In the collective setting we have:* $\boldsymbol{\xi}_{y_i} = \theta_i \boldsymbol{\xi}_{y_1}$ *for all* $i \geq 2$.

Thus the responses to different distribution factors are co-linear; this is very simple to test (see Bourguignon et al (1993)). The extra distribution factors do not, however, impose any more restrictions on the Pseudo-Slutsky matrix S. The testing of restrictions in proposition 6 constitute an independent series of tests of the collective model (which can be applied in the non-price context) to those developed in the previous section. Thus we can test for distribution factor proportionality (proposition 6) and for SR1 (proposition 1) independently. If neither is rejected then we can test for distribution factor linearity (proposition 5) with both **SR1** and distribution factor proportionality imposed. This is the route we shall follow in our empirical work below.

3.3. Restricting the dependence of distribution on prices.

We can also impose alternative structure on the distribution function μ. For example, suppose that we restrict prices to enter μ only through a known linear homogeneous price index π (**p**). This assumption smacks of *ad hocery* but it does cut down on the way price variation can affect demands a great deal. This case is particularly interesting if all of the distribution factors are money variables since in this case we can normalise and make all monetary values real. In addition, we

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can normalize prices and income in the same way. Formally, let P_i, X, Y denote real variables; i.e.,

$$P_i = \frac{p_i}{\pi(\mathbf{p})}, X = \frac{x}{\pi(\mathbf{p})}, \quad Y = \frac{y}{\pi(\mathbf{p})}$$

We then have $\mu = \mu(X, Y)$; moreover, demands can be expressed as functions of real variables :

$$\xi_i(\mathbf{p}, x, y) = \xi_i\left(\frac{\mathbf{p}}{\pi(\mathbf{p})}, \frac{x}{\pi(\mathbf{p})}, \frac{y}{\pi(\mathbf{p})}\right) = \xi_i(\mathbf{P}, X, Y)$$

Then we have that :

Proposition 7. *If there is only a single distribution factor and $\mu = \mu(X, Y)$ (with the above notations), then the Pseudo-Slutsky matrix takes the form: $S = \Gamma + k\xi_y\boldsymbol{\xi}'$ where k is a constant.*

Since the two components of the outer product on the right hand side are observable this gives an immediate test of the collective model with a known linear homogeneous price index and a single distribution factor. Note that we need to know the price index *a priori* to deflate x and y. The condition given in proposition 7 is a special case of the condition given in proposition 6 above (the vector $\mathbf{v}$ is replaced by $k\xi$).

4. A PARAMETRIC DEMAND SYSTEM

4.1. A quadratic log demand system.

In this section we take a parameterisation for the demand system and derive the implications of the restrictions implied by the collective setting. Our attention will focus on tests of symmetry and 'symmetry plus rank one' (**SR1**) and the restrictions imposed for distribution factors (propositions 5 and 6). When choosing a demand system it is important to allow for as much flexibility as possible since tests of symmetry may be biased if the parameterisation is too restrictive a priori. Thus we start with the Quadratic Almost Ideal Demand System (**QUAIDS**) of Banks, Blundell and Lewbel (1992)[9]. This system takes the **AI** demand system,

[9]The **QUAIDS** of Banks *et al* is not the only generalisation of the AI model that has this property (see, for example, the quadratic **AI** model of Fry and Pashardes (1992)) but in the absence of any evidence that any one of these is better than any other we choose to work with it.

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which includes a term in log deflated total expenditure, and adds a quadratic term in log deflated total expenditure to it. Although it might be preferable to use nonparametric methods these are not yet sufficiently developed to allow us to estimate multi-equation systems with endogenous right hand side variables and cross-equation restrictions[10]. The parameterisation chosen is, however, very flexible and admits of different shaped Engel curves even when the integrability conditions are imposed (formally, it is rank three in the sense of Lewbel (1991)). The non-parametric analysis presented in Banks *et al* (1992) suggests that this quadratic log system captures all of the significant curvature in Engel curves.

We model the budget share n-vector $\boldsymbol{\omega}$ as a function of log prices and log total expenditure. To save on notation we now take $\mathbf{p}$ to be the n-vector of *log* prices (rather than the vector of levels of prices); as before, we denote total expenditure as x. The **QUAIDS** demand system takes the vector form:

$$\boldsymbol{\omega} = \boldsymbol{\alpha} + \boldsymbol{\Gamma}\mathbf{p} + \boldsymbol{\beta}\left(\ln(x) - a(\mathbf{p})\right) + \boldsymbol{\lambda}\frac{\left(\ln(x) - a(\mathbf{p})\right)^2}{b(\mathbf{p})} \tag{4.1}$$

where , $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{\lambda}$ are n-vectors of parameters and Γ is an $n \times n$ matrix of parameters. In our empirical work below we shall allow these parameters to depend on demographics but for now we work with just prices and total expenditure. The price indices $a(\mathbf{p})$ and $b(\mathbf{p})$ are defined as:

$$a(\mathbf{p}) = \alpha_0 + \boldsymbol{\alpha}'\mathbf{p} + \frac{1}{2}\mathbf{p}'\Gamma\mathbf{p} \tag{4.2}$$

and

$$b(\mathbf{p}) = \exp(\boldsymbol{\beta}'\mathbf{p}) \tag{4.3}$$

Note that 4.1 reduces to the **AI** model if the $\boldsymbol{\lambda}$ vector is zero. Adding up implies that $\boldsymbol{\alpha}'\mathbf{e} = \mathbf{1}$ and $\boldsymbol{\beta}'\mathbf{e} = \boldsymbol{\lambda}'\mathbf{e} = \boldsymbol{\Gamma}'\mathbf{e} = 0$ where $\mathbf{e}$ is an n-vector of ones. Homogeneity implies that $\boldsymbol{\Gamma}'\mathbf{e} = 0$. We shall derive the symmetry restrictions in the next subsection.

In all that follows we shall always impose homogeneity[11]. Adding up is automatically implied by the data construction. Thus we drop the last equation to accommodate adding up and work with homogeneous prices (that is, prices

[10] See Brown and Matzkin (1995) for a recent contribution along these lines.

[11] One of the more encouraging results of moving from testing on aggregate data to micro data is that homogeneity is not usually rejected. Tests for homogeneity on the data used below (not reported) also fail to reject.

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divided by the price of the good that is dropped from the system). Then we estimate the parameters of the $(n-1)$-vectors $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda})$ without their last elements and the parameters of the $(n-1) \times (n-1)$ Γ matrix without its last row and column. To cut down on notation, we now take n to be the number of goods minus one and $(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{\lambda})$ and Γ to be these reduced vectors and matrices.

We derive the Pseudo-Slutsky matrix for the parameterisation in equation 4.1 using the budget share form:

$$S = \boldsymbol{\omega}_{\mathbf{p}} + \boldsymbol{\omega}_{\mathbf{x}} \boldsymbol{\omega}'$$

where $\boldsymbol{\omega}_{\mathbf{p}}$ is the $n \times n$ Jacobian matrix of partial derivatives of the budget shares with respect to log prices and $\boldsymbol{\omega}_{\mathbf{x}}$ is the gradient of $\boldsymbol{\omega}$ with respect to $\ln x$. Applying this to 4.1 to 4.3 we have:

$$\begin{aligned} S &= \Gamma - \frac{1}{2}\left(\boldsymbol{\beta} + 2\boldsymbol{\lambda}\frac{\tilde{x}}{b(\mathbf{p})}\right)\mathbf{p}'(\Gamma - \Gamma') + \\ &\quad \tilde{x}\left(\boldsymbol{\beta}\boldsymbol{\beta}' + \frac{\tilde{x}}{b(\mathbf{p})}(\boldsymbol{\lambda}\boldsymbol{\beta}' + \boldsymbol{\beta}\boldsymbol{\lambda}') + \left(\frac{\tilde{x}}{b(\mathbf{p})}\right)^2 \boldsymbol{\lambda}\boldsymbol{\lambda}'\right) \end{aligned} \tag{4.4}$$

where $\tilde{x} = \ln(x) - a(\mathbf{p})$. Since all of the parameters in 4.4 are identified from the system 4.1 we can use this for testing.

4.2. Testing for symmetry and SR1.

We are now in a position to give the necessary and sufficient conditions for symmetry and 'symmetry plus rank one' (**SR1**) for our parameterisation.

$$$

Thus the matrix of parameters inherits the symmetry and SR1 properties of S. This makes testing relatively easy; all we need to do is test for parametric restrictions on the estimated Γ,using the conditions in Lemma 3.

4.3. Testing for other implications of the collective model.

In the demand system given in 4.1 we conditioned only on prices and total expenditure but other observable factors also have an important influence on demand patterns. Following the distinction made in section 3 we designate these other variables as either 'preference factors', $\mathbf{z}$, or 'distribution factors' $\mathbf{y}$. We include the preference factors in the conventional way by allowing them to modify the parameters of the indices $a(\mathbf{p})$ and $b(\mathbf{p})$:

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$$a(\mathbf{p},\mathbf{z}) = \alpha_0 + \boldsymbol{\alpha}(\mathbf{z})'\mathbf{p} + \frac{1}{2}\mathbf{p}'\Gamma\mathbf{p} \tag{4.5}$$

and

$$b(\mathbf{p},\mathbf{z}) = \exp\left(\boldsymbol{\beta}(\mathbf{z})'\mathbf{p}\right) \tag{4.6}$$

In our parameterisation we take $\boldsymbol{\alpha}(\mathbf{z})$ and $\boldsymbol{\beta}(\mathbf{z})$ to be linear; that is:

$$\boldsymbol{\alpha}(\mathbf{z}) = \boldsymbol{\alpha}^0 + \boldsymbol{\alpha}^1 z_1 + ...\boldsymbol{\alpha}^{l_\alpha} z_{l_\alpha} \tag{4.7}$$

where l_α is the number of preference factors included in the $\alpha(.)$ term and the α^k's are n-vectors. Similarly we have:

$$\boldsymbol{\beta}(\mathbf{z}) = \boldsymbol{\beta}^0 + \boldsymbol{\beta}^1 z_1 + ...\boldsymbol{\beta}^{l_\beta} z_{l_\beta} \tag{4.8}$$

where l_β is not necessarily equal to l_α.

Note that in 4.5 we follow most other investigators and assume that the price response terms are the same for all households within any given strata. It is important to emphasise, however, that in our empirical work below we stratify fairly finely and estimate separate demand systems for different strata. Thus we only impose that price responses are the same within strata and not across the whole population. In particular, we shall allow the matrix Γ to vary across households of different sizes. In the present context, imposing that Γ is the same across single people and couples would be particularly inappropriate since the former should have a symmetric Γ, whereas the latter may not (unless the unitary model holds for couples).

To incorporate the distribution factors we note that propositions 5 and 6 refer to the derivatives of demand with respect to such factors. Thus it is convenient to include these in the constant term in 4.1:

$$\boldsymbol{\omega} = \boldsymbol{\alpha}(\mathbf{z}) + \boldsymbol{\Theta}\mathbf{y} + \Gamma\mathbf{p} + \boldsymbol{\beta}(\mathbf{z})(\ln(x) - a(\mathbf{p},\mathbf{z})) + \boldsymbol{\lambda}\frac{(\ln(x) - a(\mathbf{p},\mathbf{z}))^2}{b(\mathbf{p},\mathbf{z})} \tag{4.9}$$

where $\mathbf{y}$ is an m-vector of distribution factors and Θ is an $n \times m$ matrix of parameters. We denote the kth. column of Θ by Θ^k.

The next condition we are interested in testing is the distribution factor proportionality condition given in proposition 6. For our parameterisation this is

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equivalent to Θ having rank 1. This is most easily tested by testing for the following condition on the columns of Θ:

$$\Theta^k = \tau_k \Theta^1 \text{ for } k \geq 2 \tag{4.10}$$

If this condition and SR1are not rejected then we can go on to test distribution factor linearity (see proposition 5). This states that the (observable) vector of the derivatives of demand with respect to the factor **y** be a linear combination of the first two columns of the matrix M. Denoting the ith. column of M as M^i we have the following joint test for distribution factor proportionality and linearity:

$$\Theta = \begin{pmatrix} M^1 & M^2 \end{pmatrix} * \begin{pmatrix} \lambda_1 \boldsymbol{\tau} \\ \lambda_2 \boldsymbol{\tau} \end{pmatrix} \text{ where } \boldsymbol{\tau} = \begin{pmatrix} 1 & \tau_2 & . & . & . & \tau_m \end{pmatrix} \tag{4.11}$$

This restriction has $m(n-2)-1$ degrees of freedom.

In this section we have presented a flexible demand system (4.9) and a series of tests of conditions implied by the unitary and collective model. These are tests for 'symmetry' and 'symmetry plus rank 1'; 'distribution factor proportionality' and 'distribution factor linearity and proportionality'. We turn now to testing these conditions on individual household data.

5. EVIDENCE FROM THE CANADIAN FAMEX

5.1. A description of the data.

To test and estimate the collective model we need several features in the data. First, we of course need information on (household) demands; thus we have to use household data. We also need enough price variation to allow us to estimate the price responses reliably. This already rules out many data sets since this requires either a long time series of cross sections or a shorter time series with some observable cross-section price dispersion within the period. Finally we need reliable information on the individual incomes of the members of the household since these will be our prime candidates for distribution factors. We use the Canadian Family Expenditure Survey (FAMEX) which is a survey of annual purchases by households (see the Data Appendix for details). The FAMEX is not run every year; here we use the surveys for the years 1974, 1978, 1982, 1984, 1986, 1990 and 1992[12]. If intertemporal variation was the only source of relative price variation then this would not be enough years to estimate price effects; fortunately,

[12]These are all publicly available. The only other public use tape available is for 1969. We do not use the 1969 data since the price data associated with them are unreliable.

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however, there is also significant price variation within Canada (due to different provincial tax rates and transport costs) so that we can estimate reliable price responses even when we allow for cross-country taste differences.

We consider only single males; single females and couples with no one else in the household. Our primary interest is in many person households but the singles are an important control for at least two reasons. First, the demands for singles should satisfy the usual Slutsky conditions. If they do not, then it plausible that the usual rejections of the integrability conditions is due to something other than inappropriate aggregation across household members. Second, for singles we can test for the presence of different variables in demands and use this analysis in the framing of the specification for couples. For example, we find that we can exclude income from the demands for singles; this justifies taking household income and individual incomes as instruments for the unitary model for couples[13].

For couples we model the demand for eight non-durables: food at home; food outside the home; household operations (sometimes referred to as services); men's clothing; women's clothing; transport (excluding the purchase of vehicles); recreation and vices (tobacco and alcohol). For singles we model one less good since purchases of women's (men's) clothing by single men (respectively, women) are not recorded. Precise details of sample selection and variable construction and description are given in the Data Appendix. One notable feature of these data is that since the FAMEX is a survey of annual purchases there are far fewer zeros for goods such as clothing, vices and eating out than one finds in surveys based on short diaries.

We assume that the preferences for these goods are separable from all other goods except labour force status, car ownership and home ownership. We allow for non-separabilities between goods and leisure by conditioning on labour force status (see Browning and Meghir (1991)); specifically , we select on all agents being in full time employment (defined as at least 48 weeks of full time work in the survey year). We allow for the dependence of demands on car and home ownership by including dummy variables for these in our set of preference factors. Two issues arise here. First, demands may not be exogenous to these choices (or even to the selection on being single or married with no one else in the household). We shall

[13]A referee has suggested that this may not be valid if, for example, one person responds to the survey in the two person household and he or she systematically mis-reports the other person's expenditures and income and these reporting errors are correlated. Although the income information in our data is unusually reliable (see the Data Appendix) this remains a possibility that it is difficult to control for without information on who responds to the survey.

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simply assume that they are (primarily for want of decent instruments) but this is an important area for future work. The second issue is that home ownership and labour force status may be distribution factors. As discussed above we partition demographics and income variables between preference factors and distribution factors. We do this since, as can be seen from the specification in equation 4.9 we cannot separately identify the parameters for a variable that enters one or other of the utility functions and the distribution function. Thus we choose to treat all variables that enter the demands for singles (in particular, car and home ownership and labour force status) as preference factors for couples. The issue of which demographics enter the distribution function assumes a larger importance when we come to identifying 'who gets what' in the household; once again this left for future work.

5.2. Econometric issues.

Before presenting estimates of the parameters of equation 4.9 we have to address some econometric issues. First, we must allow for unobservable heterogeneity. Although it would be desirable to derive the stochastic formulation by allowing for heterogeneity in each partners preferences and the distribution function we follow usual practice and simply add a (heteroscadestic) error term to each equation.

We also allow for the possible endogeneity of total expenditure. Since the tests of the validity of these instruments plays an important role in what follows we present a preliminary discussion here; the precise details of included and excluded variables is given below. The usual reason for assuming that total expenditure might be endogenous in a demand system is that unusually high (or low) expenditure on one good by a particular household will affect both the error for that household and total expenditure; thus infrequency (or lumpiness of purchases) will induce a correlation between total expenditure and the errors in the system. Measurement error for individual expenditures also induces such a correlation. The usual instrument suggested to correct for this is net income. This is correlated with total expenditure but is usually assumed to be uncorrelated with any infrequency of purchase or measurement error. The critical point here is that within the unitary model, income should not affect demand once we condition on total expenditure. Thus it should be excluded from the right hand side of the system and is available as an instrument. The same applies to the individual incomes of the two members in the couples households. We shall return to this issue in the next sub-section in which we present a detailed account of our empirical

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specification.

The final difficulty in estimating equation 4.9 is that it is non-linear. Note, however, that if we have estimates of the indices $a(\mathbf{p}, z)$ and $b(\mathbf{p}, z)$ in 4.5 and 4.6 then we can estimate 4.9 as a system of linear equations. The obvious estimates of $a(.)$ and $b(.)$ to use are the values constructed using estimates of the $\boldsymbol{\alpha}$, Γ and $\boldsymbol{\beta}$ in the definitions of these indices. These in turn can be derived from estimates of the system. Thus we only need starting estimates of the $a(.)$ and $b(.)$ indices; we use a Stone price index for the linear homogeneous $a(.)$ and unity for the zero homogeneous $b(.)$[14]. This 'iterated moment' estimator is discussed more fully in Browning and Meghir (1991) and Blundell and Robin (1992). In practice, it works well and usually converges after three or four iterations. The only parameter that cannot be estimated in this way is α_0 in the $a(.)$ index; although it is formally identified, it is not well-determined and the final results are insensitive to the value of this parameter so we simply hold it constant in all that follows.

The tests of the conditions given in the last section are all performed using minimum chi-squared methods (see Browning and Meghir (1991) for an account of min-χ^2 tests in this context). Thus we first estimate the parameters and covariance matrix of the parameters of the system 4.9 with no restrictions using conventional GMM methods; denote these by φ and C respectively. Then we impose the restrictions by solving:

$$\min_{\eta} \left(\varphi - f\left(\eta\right)\right)' C^{-1} \left(\varphi - f\left(\eta\right)\right) \tag{5.1}$$

where $f(\eta)$ is the mapping from the restricted parameters η to the unrestricted parameters φ . The value of this minimand gives the χ^2 statistic for the restriction. The covariance matrix for the restricted parameter estimates is given by $(F'CF)^{-1}$ where F is the Jacobian of $f\left(.\right)$ evaluated at $\hat{\eta}$, the vector that minimises 5.1.

5.3. The unitary model.

We first present a conventional demand analysis for the three strata (couples, single females, single males). That is, an analysis assuming that the unitary model holds for all households. The purpose of this is to illustrate some of the problems that motivated the analysis presented in this paper. To do this we estimate the parameters of the system given in 4.9 without the Θ matrix.

[14]We tried very many other starting values; in all cases the system converged to the same estimates.

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For the singles we include 13 preference variables in the $a(.)$ index (that is, l_a in 4.7 equals 13). These are dummies for four regions of residence (Atlantic region; Quebec; Prairies and British Columbia with Ontario as the excluded region), car ownership, home ownership, living in a city, having more than high school education, white collar occupation, the respondent's mother tongue being French, the respondent's mother tongue being something other than French or English as well as age and age squared. We also allow for two variables in the $b(.)$ index: car ownership and home ownership (this choice is the result of a preliminary investigation which is not reported here). This gives 24 parameters per equation (the intercepts and variables in the $a(.)$ and $b(.)$ indices, the six homogeneous prices and the λ parameter).

The instruments for the singles are the intercept; the 13 preference factors included in the $a(.)$ index; the six log homogeneous prices; the log (absolute) price of the numeraire good and log net income; log net income squared and log net income crossed with the car and home ownership dummies. The absolute price of the numeraire good can be excluded from the demand system if homogeneity is maintained and it should also be correlated with total expenditure if agents are at all sensitive to real interest rates. As to the income variables: as discussed above, in a unitary model income should not affect demands once we condition on total expenditure but it is obviously correlated with total expenditure. One objection to this is that preferences may be correlated with demand if, for example, higher paid jobs require more expensive clothing. In this case we would expect to see that higher paid individuals have a higher budget share for clothing than lower paid individuals with the same total expenditure. This is entirely plausible, but it is also testable since we have one over-identifying restriction per equation for a total of six degrees of freedom for the system[15].

For couples we include 15 preference factors in the $a(.)$ index; this is the end result of some preliminary analysis which excluded some variables (such as the wife's language) which were found to be wholly 'insignificant' everywhere. We include 12 dummy variables and 3 continuous variables. The dummies are for: region of residence (four dummies, as for the singles); home ownership; living in a city; car ownership ; the husband having more than high school education; the husband having a white collar job; the wife having a white collar job and the husband's two language options . The three continuous variables are the age and age squared of the husband and the age of the wife. For the preference factors

[15]On the other hand, if the excluded absolute price of the numeriare does not have much explanatory power then this test is not very powerful.

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in the $b(.)$ index, we include the same variables as for singles, that is, dummy variables for car and home ownership. Thus we have 27 parameters per equation (recall that we have one more (clothing) good for couples and hence one extra price).

The instrument set for total expenditure for the couples sample includes the 15 variables included in the $a(.)$ index; the seven log homogeneous prices; the log absolute price of the numeraire good and a set of income variables. The specific income variables we use in the instrument set are also the result of a preliminary investigation which is not reported here. The main criterion for inclusion in the instrument set is that we do not want to include variables that have little explanatory power in the auxiliary equation since this simply reduces the power of the over-identifying test. In all we use six income variables: log (real) net household income; the square of log net income; log net income crossed with dummies for car ownership and home ownership; the log of the wife's gross earnings and the log of the husband's earnings. In all we have 30 instruments per equation (the intercept, fifteen preference factors; seven log homogeneous prices; the log price of the numeraire good and the six income variables). This give four over-identifying restrictions per equation and a total of 24 degrees of freedom for the six good system.

To save on space we do not present the full set of parameter estimates here[16]; rather, in Table 1 we present the tests for symmetry and for the validity of the over-identifying restrictions for our three strata.

Table 1. Tests of the unitary model restrictions.

Test for:	SINGLE FEMALES # = 2173	SINGLE MALES # = 2044	COUPLES # = 2428
Overidentification	1.9 (6) [92.6%]	6.69 (6) [35.1%]	41.8 (21) [0.54%]
Symmetry	11.1 (15) [74.7%]	17.4 (15) [29.7%]	49.4 (21) [0.05%]

Note: χ^2 test statistic; (degrees of freedom); [probability under the null].

The results for the two single strata do not display any signs of misspecification; it seems that the singles data are consistent with the unitary model (or at least the

[16]In the Appendix we present estimates for the collective model for the couples sample; all detailed results are available on request to the authors.

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implications of symmetry and the exclusion of income). The results for couples are representative of the results usually presented in the literature on demand analysis on micro data: the symmetry and the over-identifying restrictions are both rejected at conventional sizes. One reaction to this is to adjust significance levels so that we do not interpret these test statistics as indicating rejection. For example, if we use a 'Schwarz' critical level of (degrees of freedom $*$ ln (sample size)) $= 163.7$ for both the tests given here, then we would conclude that the unitary model is, *a posteriori*, the more likely. Under this interpretation there are no problems with the application of the unitary model to household data. The converse view (which is the one we take) is that the restrictions are suspect and that we cannot necessarily apply the unitary model to two person households. We now turn to testing the implications of our proposed alternative for couples, the collective model.

5.4. The collective model.

The results presented in Table 1 suggest that there are some problems with imposing the unitary model on the couples data that do not appear for singles. Thus we now estimate the collective model for couples. To do this we include two extra variables on the right hand side of the demand equations: the log of the wife's earnings minus the log of the husband's income ('the income difference') and the wife's gross income, see 4.9. We present the parameter estimates for the unrestricted demand system in Table 2; the tests of particular interest are presented in Table 3.

As can be seen, the test for the over-identifying restrictions is much improved; thus it seems that the individual incomes should be included in the demand system. The next row of Table 3 presents direct evidence on this: this is a test for excluding the two income measures from the system (see Table 2 for the individual estimates). We conclude that individual incomes are important in the demands of couples. Referring back to Table 1 we see that this is not the case for singles since income is one of the excluded variables used to identify the model and the over-identification restrictions are not rejected for singles.

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Table 3: Tests of the collective model restrictions.	
Test for:	
Over-identification.	12.2 (7) [9.3%]
Exclusion of the individual income variables.	25.9 (14) [2.7%]
Symmetry.	42.0 (21) [0.41%]
SR1.	10.0 (10) [44.3%]
Distribution factor proportionality.	7.7 (6) [26.0]
SR1, distribution factor proportionality and linearity.	27.4 (21) [15.7%]

The next two rows in Table 3 test for symmetry and 'symmetry plus rank one'. Comparing the test statistics for symmetry in Tables 1 and 2 we see that adding the individual income variables decreases the test statistic a little but not to the point where we would not reject symmetry at conventional levels of significance. The SR1 condition, however, is not rejected. Thus the price responses are consistent with the collective model.

The next row presents the test for distribution factor proportionality. As already discussed this restriction is independent of the test for SR1. The proportionality test does not reject. Finally, then, we can go on to testing for SR1, distribution factor proportionality and distribution factor linearity together, see the final row of Table 3. As can be seen these restrictions are not rejected . We conclude that the data are consistent with the collective setting.

5.5. Substantive implications of the parameter estimates.

Although the foregoing analysis indicates that we do need to weaken the unitary model for two person households it is not so clear that this has any strong im-

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plications for the values that we are usually concerned with in demand models. Specifically, what happens to total expenditure and own price elasticities if we impose the various restrictions given by the unitary and collective models? In our investigation of this, we shall impose one further restriction on our estimates of the collective model. This restriction is that it is only the difference in log earnings that enters the sharing function. This is a very natural assumption to test for in this context. The $\chi^2(1)$ value that the proportionality factor in the collective-restricted model (the last row of Table 3) is zero is 1.03; thus we can reject the hypothesis that the wife's income has a role to play over and above its effect on the differences in incomes. In all that follows we shall compare the unrestricted unitary model with the unrestricted collective model with two sharing factors (see Table 2) and the restricted collective model with only the difference in log income (see Table 4).

Before comparing the predictions from the different models we examine how demands change as the income share of the wife changes. Referring to Table 4, we see that an increase in the wife's share of income (holding everything else constant) significantly increases the demand for women's clothing and significantly decreases the demand for men's clothing and food at home. If we increase the wife's share of income from 10% to 90% (both values are within the range of our data) then the share for food at home falls from 19.5% to 17.2%; women's clothing rises from 6.2% to 7.8% and men's clothing falls from 5.5% to 4.7%. Although not significant, such a change also gives a fall in the vices budget share from 8.65% to 7.5% and a rise in the budget share for food outside the home from 10.7% to 12%.

In Tables 5 and 6 we present estimates of total expenditure elasticities and own price elasticities for three different models: the unrestricted unitary model; the unrestricted collective model with two sharing factors and the collective model with the full collective restrictions and only one sharing factor. These are evaluated for a car and home owning, English speaking couple living in a city in Ontario, both of whom are aged 40 and are in white collar work. We set total expenditure equal to median total expenditure[17] and the differences in earnings to zero.

Table 5 presents expenditure elasticities for the three different models. The most dramatic across columns is that when we include the earnings variables in the demands (column 1 to column 2), the expenditure elasticity for food at home

[17]Much the same qualitative results emerge at other points of the total expenditure distribution.

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becomes negative. This is a real surprise even though the earnings variables are highly correlated with total expenditure and might be expected to have a sizable impact on expenditure elasticities. On the other hand, once we impose the full collective conditions the expenditure elasticity for food at home becomes positive (albeit 'insignificant'). This pattern that the full collective elasticities are closer to the unrestricted unitary estimates than they are to the unrestricted collective estimates is also seen in other goods, notably recreation and vices[18]. Referring to Table 2 we see that the wife's earnings are most 'significant' for food at home, receration and vices - it is this that gives the variations across the three columns.

The estimates of own price elasticities given in Table 6 also have the pattern that the estimates (and standard errors) from the restricted collective model are close to those for the unrestricted unitary model. Once again, imposing the collective restrictions gives somewhat different elasticity estimates for food at home, recreation and vices. Generally, then, we see that estimates of elasticities from the unitary model are not very different to those from the collective-restricted model. The principal differences are in the predictions concerning the effects of the intra-household distribution of earnings on demands.

6. CONCLUSIONS

In the above we presented a general characterisation of the collective model. We showed that the collective model can be completely captured by using a household utility function $u(.)$ that depends on household purchases $\mathbf{q}$ and a distribution index μ. If the latter is a constant then we have the usual unitary model. Generally, however, the function $\mu(.)$ depends on prices $\mathbf{p}$, total expenditure x and distribution factors $\mathbf{y}$. The fact that all non-preference influences have to act through this index puts strong restrictions on household behaviour. In sections 2 and 3 we presented these restrictions.

In the empirical section we estimated the parameters of a demand system and then tested for some of the predictions of the unitary and collective models. Although we made minimal assumptions in the theory section we necessarily had to make stronger assumptions in this empirical work. For example, we have assumed that preferences over the non-durables modelled are separable from other goods (except for leisure and the ownership of a house or car). We have also assumed that the labour supply decision is exogenous for the demand system.

[18]This is *not* because of the exclusion of one the earnings variables; similar results hold for the restricted collective model with two sharing factors.

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More fundamentally, we have assumed that the marriage decision is given; that is we do not control for selection in to couples or singles. Conditional on these reservations the results are unambiguous: the predictions of the unitary model are not rejected for single people but they are rejected for couples. The predictions of the collective model are not rejected by the data for couples. This encourages us that the collective setting is worth further investigation.

As mentioned in the introduction, one of the other important areas where the results presented here can be applied directly is to the joint labour supply decision of husband and wife. The theoretical results presented in section 2 and 3 have implications for such work on cross-sectional data. Since there is no cross-section variation in prices for goods we can only define a single composite commodity, consumption, and then analyse the three 'good' system for male and female labour supply and consumption. The cross-section variation in wages gives the (relative) 'price' variation that we have exploited in this paper. Referring back to the discussion following proposition 2, however, we see that without further restrictions, the collective setting does not have any implications for price responses in a three good model. Any Slutsky responses in a three good model are consistent with the collective setting. Thus the factor proportionality restrictions (see proposition 6) are the only restrictions that the collective model imposes in this context (see also Chiappori (1990)). Additional restrictions may be derived, but only under additional assumptions, typically, privateness of leisure and consumption and restrictions on preferences (see, for example, Chiappori (1988, 1992)).

The power of thinking about the collective model in terms of a distribution function is shown by the ease with which we derived the results in sections 2 and 3. Just as importantly, this way of looking at things is likely to facilitate future work that undertakes more structural analyses of household behaviour. In particular, there are important decisions that individuals make that pre-date the allocation decisions within marriage. This obviously includes the marriage decision itself but also education and human capital decisions. If the collective setting is indeed appropriate for decision making once a union is formed then the distribution function is a useful 'sufficient statistic' for the importance of these earlier decisions in the division of the gains to marriage.

It may also be the case that assuming the collective setting allows a more precise determination of empirical effects. To give an example, suppose that it is posited that changes in law governing the division of assets on divorce leads to shifts in 'power' within the household. If we have households that are observed in different policy regimes then it may be possible to incorporate a variable capturing

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these differences in environment in the distribution function. The fact that reactions to this variable are closely related to reactions to other distribution factors and to price effects means that we may be able to determine the effects of such changes more precisely. Of course, this gain in precision comes at the expense of maintaining the collective model but we regard this as being acceptable given the foregoing.

Another area that deserves systematic exploration is the use of the distribution function in the analysis of intra-household welfare. Once we accept that households do not have a single welfare index we need to allow for differences in distribution within the household. It is likely that any such extensions that maintain the collective setting will use the distribution function even though at present it is unclear how this will be achieved since the distribution function depends on the normalisation of the utility functions used.

As emphasised in the introduction we regard the collective setting as a tractable and plausible next step in the analysis of the behaviour and welfare of many person households. The implications of the collective model are significantly weaker than those of the unitary model but not so weak as to impose no restrictions on observables In this paper we have restricted attention to demand behaviour but it is clear that the collective framework can be extended to the analysis of labour supply; fertility; savings; portfolio choice and other areas of household behaviour.

A. Proofs.

Proof. *Proposition ??* .

Just note that the maximand in (2.3) is differentiable in $(\mathbf{p},x)$ and differentiable and strongly concave in $(\mathbf{q}^A, \mathbf{q}^B, \mathbf{Q})$, while the program itself is zero homogeneous. ■

Proof. *Proposition ??*

Since $\xi(\mathbf{p}, x) = \mathbf{f}(\mathbf{p}, x, \mu(\mathbf{p}, x))$ we have: $S = \xi_\mathbf{p} + \xi_\mathbf{x}\xi' = \mathbf{f}_\mathbf{p} + \mathbf{f}_\mu\mu'_\mathbf{p} + (\mathbf{f}_\mathbf{x} + \mathbf{f}_\mu\mu_\mathbf{x})\mathbf{f}' = (\mathbf{f}_\mathbf{p} + \mathbf{f}_\mathbf{x}\mathbf{f}') + \mathbf{f}_\mu(\mu_\mathbf{p} + \mu_\mathbf{x}\mathbf{f})'$. Since $\mathbf{f}(\mathbf{p}, x, \mu)$ is a conventional uncompensated demand function for fixed μ, this gives $\Gamma = (\mathbf{f}_\mathbf{p} + \mathbf{f}_\mathbf{x}\mathbf{f}')$ is symmetric and negative semi-definite. Denoting $\mathbf{u} = \mathbf{f}_\mu$ and $\mathbf{v} = (\mu_\mathbf{p} + \mu_\mathbf{x}\mathbf{f})$ we have the result given in the proposition. ■

Proof. *Lemma 1*

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If $S = \Sigma + \mathbf{u}\mathbf{v}'$ (where Σ is symmetric) is not symmetric then $\mathbf{u}$ and $\mathbf{v}$ are linearly independent, otherwise $S = \Sigma + \mathbf{u}\mathbf{v}' = \Sigma + \lambda\mathbf{v}\mathbf{v}'$ for some λ and hence S is symmetric. Thus $M = S - S' = \mathbf{u}\mathbf{v}' - \mathbf{v}\mathbf{u}'$, the difference of the outer product of two linearly independent vectors and hence M has rank 2. Finally let $\mathbf{w}$ be in the image space of M; that is, for some $\mathbf{z}$ we have $\mathbf{w} = M\mathbf{z} = (\mathbf{u}\mathbf{v}' - \mathbf{v}\mathbf{u}')\mathbf{w} = (\mathbf{v}'\mathbf{w})\mathbf{u} - (\mathbf{u}'\mathbf{w})\mathbf{v}$ and hence $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$.

Conversely, take any antisymmetric matrix M of rank 2. Rank 2 implies that $M = \mathbf{a}\mathbf{b}' + \mathbf{c}\mathbf{d}'$ for some vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$; then antisymmetry requires that $M = \mathbf{a}\mathbf{b}' - \mathbf{b}\mathbf{a}'$ where, as above, $\mathbf{a}$ and $\mathbf{b}$ belong to $\operatorname{Im} M$. Since the latter is of dimension 2, any two vectors $\bar{\mathbf{u}}$ and $\bar{\mathbf{v}}$ can be written as :

$$\begin{aligned} \bar{\mathbf{u}} &= \alpha\mathbf{a} + \beta\mathbf{b} \\ \bar{\mathbf{v}} &= \gamma\mathbf{a} + \delta\mathbf{b} \end{aligned}$$

Then

$$\bar{\mathbf{u}}\bar{\mathbf{v}}' - \bar{\mathbf{v}}\bar{\mathbf{u}}' = (\alpha\delta - \beta\gamma)\,(\mathbf{a}\mathbf{b}' - \mathbf{b}\mathbf{a}')$$

Here, $\alpha\delta - \beta\gamma \neq 0$, for otherwise $\bar{\mathbf{u}}$ and $\bar{\mathbf{v}}$ would be colinear. For $\lambda = \frac{1}{\alpha\delta - \beta\gamma}$, we have that $M = \mathbf{a}\mathbf{b}' - \mathbf{b}\mathbf{a}' = \lambda\,(\bar{\mathbf{u}}\bar{\mathbf{v}}' - \bar{\mathbf{v}}\bar{\mathbf{u}}')$ ■

Proof. *Lemma 2.*

Let λ be an arbitrary eigenvalue, $\bar{\lambda}$ its conjugate, and z (resp. $\bar{z}$) the corresponding eigenvectors :

$$Mz = \lambda z \Leftrightarrow M\bar{z} = \bar{\lambda}\bar{z}$$

Then

$$\bar{z}'Mz = \lambda\bar{z}'z = (M'\bar{z})'z = -\bar{\lambda}\bar{z}'z$$

Since $\bar{z}'z = \|z\|^2 \neq 0$, we have that $\bar{\lambda} = -\lambda$ and λ is imaginary. Since imaginary roots come by conjugate pairs, the number of non zero eigenvalues must be even. ■

Proof. *Lemma 3*

Let M be an antisymmetric matrix with m_{12} not equal to 0. This implies that M has at least rank 2 and the first two rows of M are linearly independent.

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If M has rank 2 then the i th row of M can be written $m^i = \pi m^1 + \kappa m^2$. Since M is antisymmetric we have $m_{13} = -m_{31}$ and $m_{23} = -m_{32}$ so that $\pi = -\frac{m_{2i}}{m_{12}}$ and $\kappa = -\frac{m_{1i}}{m_{12}}$. This gives $m_{1k} = \pi m_{1k} + \kappa m_{2k} = \frac{m_{1i}m_{2k} - m_{1k}m_{2i}}{m_{12}}$ for all (i,k) such that $k > i > 2$.

Conversely, if the relationship given in the lemma holds then we can write row i for $i > 2$ as $\frac{m_{13}m^2 - m_{23}m^1}{m_{12}}$ and hence M has rank 2. ■

Proof. *Proposition 1.*

Only the final statement is new. But for any vector $\mathbf{w}$ orthogonal to $\mathrm{Im}(M)$, we have $\mathbf{w}'S\mathbf{w} = \mathbf{w}'\Sigma\mathbf{w} \leq \mathbf{0}$, since Σ is negative semi-definite. ■

Proof. *Proposition 2.*

¿From Lemma 1, S satisfies SR1 iff $M = S - S'$ is of rank zero or two. But M is antisymmetric; from Lemma 2, its rank must be even. It follows that, if $n \leq 3$, the $(n \times n)$ matrix M cannot be of rank more than two, so that SR1 is fulfilled.

Assume, now, that $n = 4$. Then M can be of rank zero, two or four. But homogeneity plus adding-up implies that $M.p = 0$, so that M cannot be of full rank. Hence, it can only be of rank zero or two and SR1 is fulfilled. ■

Proof. *Proposition 5*

1. From the proof of proposition ?? we have that $S = \Sigma + \mathbf{f}_\mu(\mu_\mathbf{p} + \mu_\mathbf{x}\mathbf{q})'$. From $\boldsymbol{\xi}(\mathbf{p}, x, y) = \mathbf{f}(\mathbf{p}, x, \mu(\mathbf{p}, x, y))$ we have $\boldsymbol{\xi}_y(\mathbf{p}, x, y) = \mathbf{f}_\mu \mu_\mathbf{y}$. Thus $S = \Sigma + \boldsymbol{\xi}_y(\mu_\mathbf{p} + \mu_\mathbf{x}\mathbf{q})'\left(\frac{1}{\mu_\mathbf{y}}\right) = \Sigma + \boldsymbol{\xi}_y\mathbf{v}'$.

2. If $M = S - S'$ has rank 2 then $\boldsymbol{\xi}_y$ and $\mathbf{v}$ in part 1 are linearly independent. Take any vector $\mathbf{w}$ that is orthogonal to $\boldsymbol{\xi}_y$ but not to $\mathbf{v}$. Then $M\mathbf{w} = \boldsymbol{\xi}_y\mathbf{v}'\mathbf{w}$ so that $\boldsymbol{\xi}_y$ is in the column space of M. ■

Proof. *Proposition 6*

¿From $\boldsymbol{\xi}(\mathbf{p}, x, y_1, y_2 \cdots y_m) = \mathbf{f}(\mathbf{p}, x, \mu(\mathbf{p}, x, y_1, y_2 \cdots y_m))$ we have $\boldsymbol{\xi}_{y_i} = \mathbf{f}_\mu \boldsymbol{\xi}_{y_1} = \frac{\mu_{\mathbf{y}_i}}{\mu_{\mathbf{y}_1}} \boldsymbol{\xi}_{y_1}$. ■

Proof. *Proposition 7*

Consider the vector $\mathbf{v}$ in Proposition 2 as a function of $(\mathbf{P}, X, Y)$. Then since $\frac{\partial \mu}{\partial P_j} = 0$, $\mathbf{v}$ is colinear to $\boldsymbol{\xi}$. ■

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Proof. *Proposition* ??

¿From equation 4.4 we have that S takes the form $S = \Gamma + \mathbf{R}(\Gamma - \Gamma') + \Sigma$ where the matrix $\mathbf{R} = \frac{1}{2}\left(\beta + 2\boldsymbol{\lambda}\frac{x}{b(\mathbf{p})}\right)\mathbf{p}'$ and Σ is symmetric.

If S is SR1 for all $(\mathbf{p}, x)$ then set prices equal to unity so that $\mathbf{p} = \mathbf{0}$ and $\mathbf{R} = \mathbf{0}$. Then $S = \Gamma + \Sigma$, which implies that Γ is SR1.

Conversely, if Γ is SR1 then we can write it as $\Gamma = \Sigma^* + \mathbf{u}\mathbf{v}'$ where Σ^* is symmetric. Then:

$M = S - S' = (I - \mathbf{R})(\mathbf{u}\mathbf{v}' - \mathbf{v}\mathbf{u}') - (\mathbf{u}\mathbf{v}' - \mathbf{v}\mathbf{u}')\mathbf{R}'$. Since $\mathbf{R}$ has at most rank 1, $(\mathbf{u}\mathbf{v}' - \mathbf{v}\mathbf{u}')\mathbf{R}'$ has at most rank 1. Thus M is the sum of matrices with at most rank 2 and 1 respectively so that it has at most rank 3. Since it is antisymmetric, by lemma 2 it has rank 0 or 2, consequently S is SR1, from lemma 1. ■

B. Data Appendix.

The Canadian FAMEX is a multistaged stratified clustered survey that collects information on annual expenditures, incomes, labour supply and demographics for individual households. The survey is run in the Spring after the survey year (that is, the information for 1978 was collected in Spring 1979). All of the information is collected by interview so that the expenditure and income data are subject to recall bias. Although this may give rise to problems, the FAMEX surveying method has the great advantage that information on annual expenditures is collected. Thus the FAMEX has much less problem with infrequency bias than do surveys based on short diaries. For example, the proportion of households reporting zero expenditure on clothing is about 3% in the FAMEX whereas it is over 50% in the U.K. FES. It is also the case that since the survey year coincides with the tax year (January to December) the income information is thought to be unusually reliable since it is collected at about the time that Canadians are filing their (individual) tax returns. These are often explicitly referenced by the enumerators.

Prices are taken from Statistics Canada. When composite commodities are created, the new composite commodity price is the weighted geometric mean of the component prices with budget shares averaged across the strata (couples, single males and single females) for weights. Thus, the weights are not the individual household budget shares.

Table D1 gives the sample selection path followed; the principal selection is on all agents being in full-time employment and under the age of 65. As well, we

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select on the education level being observed, net household income being positive and, for couples, gross earnings being above $2981 (in 1992 terms) (see "reasonable earnings" in the Table). Finally, in 1978 expenditures on recreational vehicles are not given separately from other spending on recreation. This lead to a small number of very high values for the latter in 1978; these have been deleted (see "reasonable expenditures" in the Table).

Table D1: Sample selection			
	Single females	Single males	Couples
Full sample	7,343	4,653	12,237
In full-time employment	2,229	2,084	2,512
Age < 65	2,179	2,052	2,458
Incomes positive	2,179	2,051	2,449
Education level given	2,173	2,048	2,442
Reasonable expenditures	2,173	2,044	2,440
Reasonable earnings	2,173	2,044	2,428
Sample years: 1974, 1978, 1982, 1984, 1986, 1990, 1992.			

Experiments were also made with 'cleaner' samples than those reported (for example, households with very low net incomes or high budget shares for some goods were excluded). In no case were the qualitative results different.

The next two Tables present sample means and other statistics for all of the variables used in the analysis (except for the homogeneous prices).

TABLE D2: DESCRIPTION OF BUDGET SHARES AND INCOMES.						
	COUPLES		SINGLE FEMALES		SINGLE MALES	
BUDGET SHARES	MEAN	# ZEROS	MEAN	# ZEROS	MEAN	# ZEROS
FOOD AT HOME (F)	.202	1	.205	15	.174	37
FOOD OUTSIDE (E)	.104	27	.104	74	.148	58
MEN'S CLOTHING (M)	.054	14	0	2,173	.085	23
WOMEN'S CLOTHING (W)	.084	6	.149	6	0	2,044
HHOLD OPERATIONS (H)	.125	1	.169	0	.101	1
RECREATION (R)	.107	10	.098	46	.123	33
TRANSPORT (T)	.245	9	.209	11	.247	21
VICES (V)	.078	79	.065	266	.122	121
	MEAN	STD. DEV.	MEAN	STD. DEV.	MEAN	STD. DEV.
TOTAL EXPENDITURE*	23,815	8,501	12,162	5,004	15,027	6,357
HHOLD NET INCOME*	55,324	19,248	26,039	10,101	31,478	14,192
GROSS EARNINGS (HUSB.)*	41,262	20,015	-		-	
GROSS EARNINGS (WIFE)*	29,318	13,201	-		-	
	MEAN	RANGE	MEAN	RANGE	MEAN	RANGE
PRICE OF VICES	0.53	0.175-1.04	0.52	0.175-1.04	0.54	0.175-1.04
* All values in 1992 Canadian dollars ($1 Canadian ≃ $0.75 U.S. ≃ £0.50 U.K.).						

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TABLE D3: MEANS OF DEMOGRAPHIC VARIABLES

	COUPLES	SINGLE FEMALES	SINGLE MALES
ATLANTIC	.145	.150	.137
QUEBEC	.192	.188	.169
PRAIRIES	.294	.320	.312
B.C.	.103	.100	.131
CAR OWNER	.949	.634	.773
HOMEOWNER	.645	.228	.281
CITY DWELLER	.810	.845	.811
AGE*	37.3	38.6	36.6
MORE THAN HIGH SCHOOL*	.190	.174	.226
FRANCOPHONE*	.196	.187	.172
ALLOPHONE*	.112	.095	.113
WHITE COLLAR*	.376	.406	.376
AGE OF WIFE	35.0	-	-
WIFE WHITE COLLAR	.345	-	-

* Refers to husband for couples.

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C. Tables.

Table A1. Parameter estimates for collective model.

filler	F	H	R	E	M	W	V
INTERCEP	173.22	117.33	-77.75	-31.54	-0.14	-28.06	105.81
	(79.38)	(59.46)	(59.66)	(46.15)	(27.06)	(31.43)	(53.93)
ATLANTIC	-0.19	0.55	-1.30	0.37	-0.93	0.24	0.85
	(0.88)	(0.48)	(0.79)	(0.65)	(0.33)	(0.47)	(0.71)
QUEBEC	1.59	-0.50	-0.78	0.34	-0.42	0.07	-0.40
	(0.91)	(0.55)	(0.82)	(0.69)	(0.36)	(0.50)	(0.73)
PRAIRIES	-0.78	0.67	0.12	0.97	-0.66	0.40	-2.71
	(0.89)	(0.56)	(0.85)	(0.72)	(0.41)	(0.55)	(0.72)
B.C.	-1.41	-0.32	1.09	2.31	-0.72	0.56	-2.22
	(0.94)	(0.51)	(0.88)	(0.71)	(0.36)	(0.54)	(0.75)
CAR-OWNE	-26.29	11.39	-18.21	19.55	15.16	10.27	-25.39
	(31.45)	(16.36)	(18.00)	(18.38)	(11.42)	(12.37)	(21.01)
HOME-OWN	28.93	0.55	-29.03	17.77	0.23	-3.09	-0.28
	(12.54)	(6.79)	(9.66)	(8.86)	(4.62)	(5.86)	(9.53)
CITY-DWE	0.27	-0.70	-1.04	1.70	-0.26	-0.32	0.06
	(0.49)	(0.26)	(0.46)	(0.34)	(0.18)	(0.25)	(0.37)
HUSBANDS	4.02	0.87	-8.82	-3.32	1.21	0.71	2.81
	(4.44)	(3.03)	(4.06)	(3.27)	(2.12)	(2.55)	(3.76)
AGE-SQUA	-40.93	-9.10	62.56	-24.22	8.14	1.83	-26.82
	(21.30)	(12.91)	(20.15)	(14.69)	(8.83)	(11.10)	(16.58)
HUSBAND-	-0.20	0.30	0.91	0.91	0.25	0.07	-0.65
	(0.48)	(0.29)	(0.50)	(0.41)	(0.23)	(0.30)	(0.38)
FRANCOPH	0.39	-0.65	-1.13	0.71	0.55	1.03	-0.13
	(0.71)	(0.42)	(0.62)	(0.55)	(0.26)	(0.42)	(0.53)
ALLOPHON	1.46	-0.11	-0.63	-0.51	0.09	0.56	-1.58
	(0.66)	(0.33)	(0.57)	(0.45)	(0.24)	(0.36)	(0.46)
HUSBAND-	-0.71	0.15	0.44	0.94	0.27	0.20	-0.29
	(0.39)	(0.22)	(0.39)	(0.31)	(0.16)	(0.24)	(0.30)
WIFE-WHI	-0.23	0.52	0.20	-0.47	0.33	0.23	-0.80
	(0.42)	(0.24)	(0.40)	(0.32)	(0.18)	(0.24)	(0.32)
WIFE'S-A	13.63	5.01	-2.74	-2.06	-4.13	-0.53	-0.41
	(4.54)	(3.07)	(4.28)	(3.39)	(2.13)	(2.72)	(4.02)
DIFF-IN-	-3.50	0.03	1.72	0.34	-0.02	0.01	2.94
	(1.57)	(0.91)	(1.41)	(1.23)	(0.77)	(0.77)	(1.24)
WIFES-LO	5.31	-0.09	-3.43	0.27	-0.31	0.53	-6.03
	(2.70)	(1.54)	(2.49)	(2.06)	(1.20)	(1.34)	(2.12)
PRICE(F)	-79.78	-66.79	80.64	-2.25	0.82	15.84	-22.66
	(51.52)	(41.71)	(42.70)	(32.55)	(18.15)	(22.84)	(39.57)
PRICE(H)	-88.87	-54.63	62.02	-5.16	-9.49	12.94	-12.92
	(44.77)	(37.55)	(38.15)	(28.38)	(16.17)	(19.95)	(34.90)
PRICE(R)	99.61	63.39	-79.30	12.02	0.87	-14.52	0.42
	(50.68)	(41.12)	(42.26)	(32.55)	(17.70)	(22.41)	(39.37)
PRICE(E)	13.76	-0.59	6.18	-8.82	-5.43	-13.97	9.02
	(5.97)	(3.11)	(5.56)	(4.36)	(2.21)	(3.18)	(4.87)
PRICE(M)	-2.34	-9.43	11.91	-20.77	6.22	0.72	-12.49
	(12.25)	(8.19)	(10.98)	(8.67)	(4.80)	(6.33)	(9.82)
PRICE(W)	-0.63	2.89	-11.59	16.47	-3.04	2.56	16.71
	(10.39)	(6.66)	(9.79)	(7.57)	(4.22)	(5.65)	(8.28)
PRICE(V)	-28.52	-18.02	23.23	-3.48	-0.96	4.08	-8.75
	(15.50)	(12.95)	(13.10)	(9.86)	(5.69)	(6.92)	(11.94)
B-INTERC	-56.37	-42.35	45.05	4.37	-2.01	6.70	-19.37
	(33.86)	(26.88)	(27.42)	(20.66)	(11.68)	(14.42)	(24.95)
B-CAR-OW	4.28	-3.46	3.67	-4.64	-3.71	-2.50	4.32
	(6.63)	(3.62)	(3.94)	(3.95)	(2.49)	(2.68)	(4.50)
B-HOME-O	-6.30	-0.18	6.24	-3.63	-0.09	0.56	0.03
	(2.66)	(1.46)	(2.06)	(1.88)	(0.97)	(1.24)	(2.02)
LAMBDA	3.13	4.60	-4.05	0.76	0.86	-0.06	2.69
	(3.39)	(2.90)	(2.87)	(2.12)	(1.29)	(1.52)	(2.61)

FIRST LINE OF NOTES
SECOND LINE OF NOTES

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Table A1. Parameter estimates for collective model.

filler	F	H	R	E	M	W	V
INTERCEP	92.40 (37.60)	51.93 (20.29)	5.31 (20.42)	-8.10 (26.64)	10.73 (11.92)	·28.43 (19.82)	131.69 (35.52)
ATLANTIC	-0.19 (0.78)	0.43 (0.41)	-0.56 (0.67)	0.11 (0.61)	-0.79 (0.31)	0.35 (0.45)	0.99 (0.64)
QUEBEC	0.96 (0.82)	-0.68 (0.49)	0.32 (0.71)	-0.06 (0.64)	-0.36 (0.32)	0.22 (0.47)	-0.27 (0.62)
PRAIRIES	-1.27 (0.73)	1.06 (0.41)	0.06 (0.72)	0.66 (0.61)	-0.58 (0.29)	0.19 (0.43)	-1.06 (0.54)
B.C.	-1.55 (0.87)	-0.28 (0.46)	0.61 (0.83)	1.96 (0.69)	-0.73 (0.33)	0.43 (0.48)	-1.60 (0.65)
CAR-OWNE	-29.84 (27.61)	0.62 (12.00)	-4.53 (14.16)	17.23 (15.96)	17.31 (9.50)	8.51 (10.20)	-14.28 (18.05)
HOME-OWN	19.93 (11.39)	-0.33 (6.03)	-21.84 (9.12)	19.44 (8.41)	1.16 (4.54)	-4.44 (5.79)	10.5[illegible] (8.89)
CITY-DWE	0.15 (0.48)	-0.74 (0.26)	-0.87 (0.46)	1.82 (0.33)	-0.26 (0.18)	-0.30 (0.25)	0.08 (0.37)
HUSBANDS	2.96 (4.37)	-1.60 (2.62)	-6.59 (3.94)	-3.84 (3.22)	1.47 (1.99)	1.03 (2.49)	2.43 (3.74)
AGE-SQUA	-20.86 (16.69)	-16.29 (9.80)	49.29 (14.60)	-24.20 (11.83)	5.87 (6.17)	6.91 (8.86)	-60.45 (13.05)
HUSBAND-	0.09 (0.43)	0.22 (0.26)	0.79 (0.47)	0.77 (0.38)	0.23 (0.21)	0.11 (0.28)	-1.09 (0.34)
FRANCOPH	0.16 (0.71)	-0.70 (0.42)	-1.03 (0.61)	0.81 (0.54)	0.62 (0.26)	1.06 (0.42)	-0.01 (0.53)
ALLOPHON	1.71 (0.64)	-0.17 (0.31)	-0.89 (0.52)	-0.49 (0.43)	0.10 (0.22)	0.61 (0.34)	-1.94 (0.42)
HUSBAND-	-0.59 (0.39)	0.18 (0.21)	0.34 (0.39)	0.93 (0.31)	0.23 (0.16)	0.24 (0.23)	-0.34 (0.30)
WIFE-WHI	-0.29 (0.39)	0.57 (0.23)	0.09 (0.39)	-0.36 (0.31)	0.34 (0.16)	0.21 (0.23)	-0.73 (0.30)
WIFE'S-A	15.09 (4.48)	7.25 (2.75)	-4.88 (4.17)	-1.73 (3.35)	-4.43 (2.05)	-0.76 (2.67)	-1.09 (3.98)
DIFF-IN-	-0.53 (0.22)	-0.26 (0.16)	0.17 (0.17)	0.29 (0.22)	-0.18 (0.08)	0.38 (0.15)	-0.26 (0.18)
PRICE(F)	-3.07 (9.38)	-12.26 (6.24)	2.63 (6.64)	-6.62 (8.35)	0.36 (3.41)	12.21 (6.28)	-29.50 (11.02)
PRICE(H)	-22.97 (10.30)	-6.05 (7.73)	4.56 (7.00)	-6.86 (8.89)	-9.92 (3.69)	13.10 (6.68)	-30.07 (11.94)
PRICE(R)	15.02 (8.55)	7.18 (5.98)	-5.73 (7.20)	10.10 (8.16)	0.41 (3.21)	-12.07 (5.70)	16.81 (9.64)
PRICE(E)	16.63 (5.86)	-1.23 (3.09)	9.26 (5.18)	-10.48 (4.52)	-6.66 (2.18)	-12.06 (3.13)	-1.89 (4.88)
PRICE(M)	7.00 (8.43)	-2.94 (4.88)	-6.05 (5.27)	-18.32 (6.82)	3.03 (3.69)	-0.76 (5.23)	-16.76 (7.27)
PRICE(W)	-5.07 (7.17)	-3.00 (4.11)	2.35 (5.28)	13.81 (5.66)	-2.05 (3.22)	4.66 (4.43)	11.17 (5.66)
PRICE(V)	-20.77 (19.73)	-20.62 (14.33)	8.00 (12.51)	-17.84 (17.76)	-16.93 (7.15)	13.31 (13.58)	-73.42 (25.80)
B-INTERC	-10.32 (13.40)	-14.99 (8.89)	1.12 (8.13)	-5.94 (11.29)	-9.06 (4.59)	8.76 (8.69)	-50.44 (16.07)
B-CAR-OW	4.79 (5.15)	-0.76 (2.29)	0.37 (2.71)	-3.72 (3.03)	-3.74 (1.83)	-1.98 (1.96)	1.91 (3.40)
B-HOME-O	-3.63 (2.12)	0.12 (1.13)	4.04 (1.71)	-3.58 (1.58)	-0.23 (0.85)	0.72 (1.09)	-2.04 (1.65)
LAMBDA	-0.69 (1.42)	1.60 (1.04)	0.17 (0.92)	1.86 (1.29)	1.62 (0.53)	-0.26 (0.99)	5.36 (1.86)

FIRST LINE OF NOTES
SECOND LINE OF NOTES

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sous la responsabilité de Pierre-André Chiappori ; Commissariat général du Plan."

Table 5. Predicted budget shares.			
Model→	Unitary	Collective	
Restriction→	Unrestricted	Unrestricted	Collective
Food at home	18.1 (0.8)	16.0 (1.1)	18.3 (0.8)
Household operations	14.0 (0.5)	13.4 (0.7)	14.2 (0.4)
Recreation	11.0 (0.7)	13.0 (1.0)	10.7 (0.7)
Food outside	10.9 (0.6)	10.2 (0.8)	11.4 (0.6)
Men's clothing	5.2 (0.3)	5.1 (0.4)	5.1 (0.3)
Women's clothing	7.0 (0.4)	7.0 (0.6)	7.0 (0.4)
Vices	8.4 (0.7)	9.0 (0.9)	8.1 (0.7)
Transport	25.4 -	26.3 -	25.2 -

Note: All values multiplied by 100. Standard errors given in brackets.

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Table 6. Total expenditure elasticities.			
Model→	Unitary	Collective	
Restriction→	Unrestricted	Unrestricted	Collective
Food at home	0.19 (0.11)	-0.68 (0.42)	0.12 (0.09)
Household operations	1.11 (0.13)	1.02 (0.25)	1.04 (0.08)
Recreation	1.53 (0.21)	2.10 (0.38)	1.68 (0.15)
Food outside	1.39 (0.16)	1.37 (0.41)	1.48 (0.13)
Men's clothing	1.64 (0.20)	1.56 (0.49)	1.65 (0.14)
Women's clothing	1.70 (0.19)	1.59 (0.37)	1.70 (0.17)
Vices	1.38 (0.22)	2.34 (0.44)	1.41 (0.21)
Transport	0.67 -	0.61 –	0.65 -

Note: Standard errors given in brackets.

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Table 7. Own price elasticities.			
Model→	Unitary	Collective	
Rrestriction→	Unrestricted	Unrestricted	Collective
Food at home	-0.45 (0.94)	+0.78 (3.84)	-0.54 (0.33)
Household operations	-1.12 (1.04)	-1.11 (2.41)	-1.00 (0.30)
Recreation	-1.14 (1.40)	-1.01 (3.38)	-1.41 (0.60)
Food outside	-1.81 (0.41)	-1.83 (0.58)	-1.52 (0.70)
Men's clothing	+0.72 (0.95)	+0.40 (0.71)	+0.46 (0.62)
Women's clothing	-0.30 (0.78)	-0.53 (0.74)	-0.10 (0.65)
Vices	-2.08 (2.61)	-1.06 (0.97)	-2.12 (2.39)
Transport	-1.85 -	-0.86 -	-1.35 -
Note: Standard errors given in brackets.			

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ANNEXE 5

L'approche par le calcul différentiel extérieur

La documentation Française : "Modèles collectifs d'offre de travail : rapport final du comité ""Emploi - travail - redistribution"" / sous la responsabilité de Pierre-André Chiappori ; Commissariat général du Plan."

Aggregation and Market Demand : an Exterior Differential Calculus Viewpoint*

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Abstract

In many economic contexts, a given function can be disaggregated as a linear combination of gradients. This problem is typical in aggregation theory, as illustrated by the literature on the characterization of aggregate demand and excess demand. We show that exterior differential calculus provides very useful tools to address these problems. In particular, we show, using these techniques, that any analytic mapping in $\mathbb{R}^n$ satisfying Walras Law can be locally decomposed as the sum of n individual, utility-maximizing market demand functions.

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1. Introduction : aggregation and gradient structures

In many situations, economists are interested in the behavior of aggregate variables that stem from the addition of several elementary demand or supply functions. In turn, each of these elementary components results from some maximizing decision process at the 'individual' level. A standard illustration is the characterization of aggregate market or excess demand in an exchange economy, a problem initially raised by Sonnenschein (1973a,b) and to which a number of author contributed, starting with Mantel (1974) and Debreu (1974), and including McFadden et al. (1974), Mantel (1976, 1977), Diewert (1977) and Geanakoplos and Polemarchakis (1980). Here, agents maximize utility under budget constraint, and individual demands add up to an aggregate demand or excess demand function. This research has recently been extended to incomplete markets by Bottazzi and Hens (1996) and Gottardi and Hens (1995). A different but related example is provided by Browning and Chiappori (1994), who consider the demand function of a two-person household, where each member is characterized by a specific utility function and decisions are only assumed to be Pareto-efficient.

These models share a common feature : they lead to the same type of mathematical problem. Specifically, in all cases, the economic context has the following translation : some given function $\mathbf{X}(\mathbf{p})$, mapping $\mathbb{R}^{\ell}_{+}$ to $\mathbb{R}^{\ell}$, can be decomposed as a linear combination of n gradients. Here, n is the number of individuals; $\mathbf{X}(\mathbf{p})$ is the original (aggregate) function; and gradients are the natural mathematical translation of the underlying optimization problem. Formally, $\mathbf{X}(\mathbf{p})$ writes down as :

$$\mathbf{X}(\mathbf{p}) = \lambda_1(\mathbf{p})\, D_{\mathbf{p}}V^1(\mathbf{p}) + \ldots + \lambda_n(\mathbf{p})\, D_{\mathbf{p}}V^n(\mathbf{p}) \qquad (1.1)$$

where the $\lambda_i(\mathbf{p})$ and the $V^i(\mathbf{p})$ are scalar functions (V^i being in general interpreted as an indirect utility function), and where $D_{\mathbf{p}}V^i(\mathbf{p})$ is the gradient of $V^i(\mathbf{p})$ at $\mathbf{p}$:

$$D_{\mathbf{p}}V^i(\mathbf{p}) = \left(\frac{\partial V^i}{\partial p_1}, \ldots, \frac{\partial V^i}{\partial p_\ell}\right)'$$

Note that, depending on the context, these functions may have to fulfill specific, additional conditions, such as positiveness, monotonicity, (quasi-)convexity, budget constraints and others.

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A natural question is then the following : what does (1.1) imply upon the form of the function **X** ?

From a mathematical point of view, the structure (1.1) is highly specific. In the first half of this century, Elie Cartan developed a set of concepts, usually referred to as exterior differential calculus (from now on EDC), that proved especially convenient to deal with problems of this type. Surprisingly enough, however, these tools have hardly ever been used in the field of economic theory. As an obvious exception, one must mention a pioneering paper by Russell and Farris (1993), showing that Gorman's rank theorem is a consequence of well-known results on Lie groups. More recently, Russell (1994) proposes a measure of 'quasi-rationality' directly based upon EDC. These works, however, only consider individual behavior[1].

The goal of this paper is to apply the tools of EDC to some standard economic aggregation problems. Specifically, we describe in some details how a very powerful theorem of EDC, due to Cartan and Kähler, can be used to address a range of issues relating to the aggregation literature. To our knowledge, this result has never been used so far in economics, although its scope potentially includes many important issues. To illustrate the latter claim, we consider the classical problem, initially raised by Sonnenschein (1993b), of the characterization of aggregate demand[2]. Two versions have been considered in the literature. The first version considers the excess demand of an exchange economy. Given some arbitrary, continuous function Z that satisfies homogeneity and Walras Law, is it possible to construct an economy (i.e., a set of preferences and initial endowments) for which Z is the aggregate excess demand ? This was solved by Mantel (1994) and Debreu (1994), and complementary results were derived by Geanakoplos and Polemarchakis (1980). The second problem is similar, except that it considers market demand (instead of excess demands). This problem, even in the local version put forth by Shafer and Sonnenschein (1982), was still open, except for the case of finite data sets (Andreu 1983).

In what follows, we provide a formal proof of the general conjecture already formulated in Sonnenchein's paper, namely that any (smooth enough) function satisfying Walras Law can be locally seen as the aggregate market demand of some economy, even when the distribution of income is imposed a priori. Our approach is general, and applies to excess demand as well - although, for the sake of brevity,

[1]A referee rightly pointed out that EDC is mentioned in Hurwicz (19), in the context of integration of consumer demand.

[2]Another application to "collective" household behavior a la Browning and Chiappori is provided in a companion paper (Chiappori and Ekeland 1997a).

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we only consider the market demand case in the paper. In addition, we provide two new results. One is specific to market demand; it shows that the result holds when the income distribution, instead of being constant, is some *given* function of prices. On the contrary, when income distribution can vary with prices in an arbitrary way, then the number of agents required is considerably decreased. The second result is valid in both the excess and market demand contexts; it states that the decomposition can be varied continuously with the initial demand function.

The structure of the paper is as follows. In the next section, we quickly recall the structure of the problems under consideration, and describe their formulation in terms of exterior differential calculus. Section 3 presents the main results, while the extensions are discussed in section 4.

2. Characterization of aggregate excess and market demand

2.1. A statement of the problem

The problem of characterizing the structure of aggregate excess and market demand was initially raised in Sonnenschein's seminal paper (1993b). The excess demand problem can be stated as follows. Take some homogeneous, smooth mapping $\mathbf{Z}(\mathbf{p}) : \mathbb{R}^{\ell} \to \mathbb{R}^{\ell}$, such that

$$\mathbf{p}.\mathbf{Z}(\mathbf{p}) = 0 \tag{2.1}$$

(Walras Law). Can we find n homogeneous, smooth individual demand functions $\mathbf{z}^1(\mathbf{p}), ..., \mathbf{z}^n(\mathbf{p})$ such that

$$\mathbf{Z}(\mathbf{p}) = \mathbf{z}^1(\mathbf{p}) + ... + \mathbf{z}^n(\mathbf{p}) \tag{2.2}$$

where $\mathbf{z}^i(\mathbf{p}) = \mathbf{x}^i(\mathbf{p}) - \omega^i$ and $\mathbf{x}^i(\mathbf{p})$ is a solution of

$$\begin{aligned} V^i(\mathbf{p}) = \max U^i(\mathbf{x}^i) \\ \mathbf{p}.\mathbf{x}^i = \mathbf{p}.\omega^i \\ \mathbf{x}^i \geq 0 \end{aligned} \tag{2.3}$$

for some well-behaved utility function U^i and some positive initial endowments $\omega^1, ..., \omega^n$?

The market demand problem is similar, except that the initial mapping $\mathbf{X}(\mathbf{p})$ should satisfy

$$\mathbf{p}.\mathbf{X}(\mathbf{p}) = n \tag{2.4}$$

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but needs not to be homogeneous, and that (2.2) and (2.3) should respectively be replaced by :

$$\mathbf{X}(\mathbf{p}) = \mathbf{x}^1(\mathbf{p}) + ... + \mathbf{x}^n(\mathbf{p}) \tag{2.5}$$

and

$$\begin{aligned} V^i(\mathbf{p}) = \max U^i(\mathbf{x}^i) \\ \mathbf{p}.\mathbf{x}^i = 1 \\ \mathbf{x}^i \geq 0 \end{aligned} \tag{2.6}$$

As it is well known, the former question - the characterization of aggregate excess demand - has been solved by Mantel (1974) and Debreu (1974), whereas the so-called "market demand" problem is still open. As a matter of fact, the techniques we shall now describe apply to both problems in basically the same way. This is partly due to the local nature of our approach. It has been known since Sonnenschein's paper that, in contrast with the excess demand problem, the characterization of market demand will face complex non-negativity restrictions. In particular, he exhibits a counter-example of a function $\mathbf{X}$ that cannot be globally decomposed as above because of these constraints. However, the local version of the problem remains : is it possible, for any given $\mathbf{p} >> 0$, to find individual demand functions $\mathbf{x}^1(\mathbf{p}), ..., \mathbf{x}^n(\mathbf{p})$, defined on some neighborhood of $\mathbf{p}$, such that (2.2) and (2.6) are fulfilled on this neighborhood ?

A result initially demonstrated by Sonnenschein (1973b) and then generalized by Diewert (1977) and Mantel (1977), states that for $n \geq \ell$ any continuous function satisfying Walras Law does, when considered *at some given point* $\bar{\mathbf{p}}$, 'look like' aggregate market demand, in the following sense : it is possible to find individual demand functions $\mathbf{x}^1(\mathbf{p}), ..., \mathbf{x}^n(\mathbf{p})$ such that :

$$\mathbf{X}(\bar{\mathbf{p}}) = \sum_i \mathbf{x}^i(\bar{\mathbf{p}}), \; D_{\mathbf{p}}\mathbf{X}(\bar{\mathbf{p}}) = \sum_i D_{\mathbf{p}}\mathbf{x}^i(\bar{\mathbf{p}})$$

In their 1982 survey, Shafer and Sonnenschein ask whether it is possible to go beyond this result, and find the $\mathbf{x}^i(\mathbf{p})$ such that $\mathbf{X}(\mathbf{p})$ coincides with $\sum_i \mathbf{x}^i(\mathbf{p})$ on an open neighborhood of $\bar{\mathbf{p}}$. While Andreu (1983) has demonstrated this property for finite sets of price-income bundles, the continuous version has not yet been established. In what follows, we show that the answer to the question is positive, at least if we assume that the function $\mathbf{X}$ is analytic on such a neighborhood (which implies, in particular, that it is infinitely differentiable).

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2.2. The basic partial differential equations

2.2.1. Excess demand

Our first basic remark is that both problems can actually be stated as partial differential equations. We start with excess demand. If V^i denote consumer i's indirect utility, we know that utility maximization implies $D_{\mathbf{p}}V^i(\mathbf{p}) = -\alpha_i.\mathbf{z}^i(\mathbf{p})$, where α_i is the Lagrange multiplier. It follows that :

$$\begin{aligned} \mathbf{Z}(\mathbf{p}) &= -\frac{1}{\alpha_1(\mathbf{p})} D_{\mathbf{p}}V^1(\mathbf{p}) - \ldots - \frac{1}{\alpha_n(\mathbf{p})} D_{\mathbf{p}}V^n(\mathbf{p}) \\ &= \lambda_1(\mathbf{p})\, D_{\mathbf{p}}V^1(\mathbf{p}) + \ldots + \lambda_n(\mathbf{p})\ D_{\mathbf{p}}V^n(\mathbf{p}) \end{aligned} \tag{2.7}$$

and $\mathbf{Z}(\mathbf{p})$ must be a linear combination of n gradients. In addition :

- the V^i are homogenous and (quasi) convex
- the λ_i are one-homogenous and negative
- furthermore, the budget constraint implies :

$$\mathbf{p}.D_{\mathbf{p}}V^i(\mathbf{p}) = 0 \quad \forall i \tag{2.8}$$

The problem is thus to find, on a neighborhood of some given $\bar{\mathbf{p}}$, functions $\lambda_1, \ldots, \lambda_n$ and $V^1, \ldots, V^n$ satisfying (2.7) and the set of conditions (2.8).

2.2.2. Market demand

The statement of the market demand problem is similar. First, if V^i denote consumer i's indirect utility, we know that utility maximization implies $D_{\mathbf{p}}V^i(\mathbf{p}) = -\alpha_i.\mathbf{x}^i(\mathbf{p})$, where α_i is the Lagrange multiplier. It follows that :

$$\begin{aligned} \mathbf{X}(\mathbf{p}) &= -\frac{1}{\alpha_1(\mathbf{p})} D_{\mathbf{p}}V^1(\mathbf{p}) - \ldots - \frac{1}{\alpha_n(\mathbf{p})} D_{\mathbf{p}}V^n(\mathbf{p}) \\ &= \lambda_1(\mathbf{p})\, D_{\mathbf{p}}V^1(\mathbf{p}) + \ldots + \lambda_n(\mathbf{p})\ D_{\mathbf{p}}V^n(\mathbf{p}) \end{aligned} \tag{2.9}$$

and $X(\mathbf{p})$ must be a linear combination of n gradients. In addition :

- the V^i are (quasi) convex and decreasing
- the λ_i are negative

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- furthermore, the budget constraint implies :

$$\mathbf{p}.D_{\mathbf{p}}V^i(\mathbf{p}) = 1/\lambda_i \quad \forall i \tag{2.10}$$

The problem is thus to find, on a neighborhood of some given $\bar{\mathbf{p}}$, functions $\lambda_1, ..., \lambda_n$ and $V^1, ..., V^n$ satisfying (2.9) and the set of conditions (2.10).

It should be noted that the two problems above are basically similar; in both cases, we have to solve a partial differential equation on a set of functions that satisfy specific constraints. The only difference lies in the form of the constraints - (2.8) in one case, (2.10) in the other - that are similar but not identical. Surprisingly enough, this minor difference results in important discrepancies in the resolution process. As we shall see, the market demand problem is much more difficult to solve than the excess demand one, even when fully exploiting the power of the EDC techniques. This may explain why the former remained unsolved for twenty five years whereas a solution of the latter was found within a few years. Also, it must be emphasized that these differences are totally independent of the nonnegativity restrictions; they are related to the mathematical nature of the (local) problem.

3. Mathematical resolution

3.1. The basic strategy

We now come to the resolution of the problems described above. Let us describe the general strategy used throughout the proofs. The basic idea is similar in both cases; the presentation is given in the case of market demand.

A first idea could be to directly consider the basic partial differential equations on $\mathbb{R}^\ell$. In the case, say, of market demand, the latter can be rewritten as : find (quasi convex) functions $V^1, ..., V^n$ that are solutions of :

$$\mathbf{X}(\mathbf{p}) = \sum_{i=1}^{n} \frac{D_{\mathbf{p}}V^i(\mathbf{p})}{\mathbf{p}.D_{\mathbf{p}}V^i(\mathbf{p})} \tag{3.1}$$

This PDE, however, does not belong to any usual class, and we do not have standard existence results to apply. The problem, thus, must be reformulated in a different way.

In fact, the basic idea of the proof is to enlarge the space under consideration, and to adopt a geometric viewpoint. Specifically, let us consider the space $E =$

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$\{\mathbf{p}, \lambda_1, ..., \lambda_n, \mathbf{\Delta}^1, ..., \mathbf{\Delta}^n\} = \mathbb{R}^{\ell+n+\ell n}$; here, the vector $\mathbf{\Delta}^i$ will later be interpreted as the gradient $D_{\mathbf{p}}V^i$. Assume, now, that a solution exists. Then the equations $\lambda_i = \lambda_i(\mathbf{p})$ and $\mathbf{\Delta}^i = \mathbf{\Delta}^i(\mathbf{p})$ define a (ℓ-dimensional) manifold $\mathcal{S}$ in E. Clearly, $\mathcal{S}$ must be included in the ℓn-dimensional manifold $\mathcal{M}$ defined by :

$$\begin{aligned} \mathbf{X}(\mathbf{p}) &= \sum_i \lambda_i \mathbf{\Delta}^i \\ \mathbf{p}.\mathbf{\Delta}^i &= 1/\lambda_i \quad \forall i \end{aligned} \tag{3.2}$$

Conversely, is it the case that *any* ℓ-dimensional submanifold of $\mathcal{M}$ defines a solution of our problem ? Certainly not. Indeed, the function $\mathbf{\Delta}^i(\mathbf{p})$ must, in addition, be the gradient of some quasi-convex function - a fact that implies well-known cross-derivative restrictions :

$$\forall i, j, \ \frac{\partial \mathbf{\Delta}^i}{\partial p_j} = \frac{\partial \mathbf{\Delta}^j}{\partial p_i} \tag{3.3}$$

plus some positivity conditions. The latter will turn out to be manageable in our local approach, since if they hold at some point, they will hold in the neighbourhood as well. If we forget them for the moment, we see that the mathematical problem can be stated as follows : *solve the system of partial differential equations (3.3) on the manifold* $\mathcal{M}$.

Conversely, assume that we can find functions $\lambda_i(\mathbf{p})$ and $\mathbf{\Delta}^i(\mathbf{p})$, that remain on the manifold, and such that the system (3.3) is satisfied. Then $\mathbf{\Delta}^i(\mathbf{p})$ is the gradient of some function V^i, and the $(V^1, ..., V^n)$ solve the problem.

In the language of EDC, the system (3.3) can simply be rewritten as :

$$\omega^i = \sum_j d\mathbf{\Delta}^{ij} \wedge dp_j = 0 \quad \forall i \tag{3.4}$$

(see Chiappori-Ekeland (1996) for a detailed explanation). Here, ω_i is a *form*, and we are looking for an n-dimensional integral manifold of the exterior differential system (3.4) on the manifold $\mathcal{M}$ . .

As it turns out, one of the most important application of EDC is precisely to provide existence theorems for partial differential equation on manifolds. Although we shall not attempt to present here EDC in details[3], we may briefly indicate the main intuitions underlying the approach from a few examples.

[3] See Bryant and al. (1991), or Chiappori and Ekeland (1996) for a pedagogical summary.

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3.2. An introductory example

3.2.1. Cauchy theorem

Let us start from the simplest version of our problem, namely the Cauchy theorem for ordinary differential equations. It states that, given a point $\bar{x} \in \mathbb{R}$ and a C^1 function f, defined from some neighborhood $\mathcal{U}$ of $(0, \bar{x})$ into $\mathbb{R}$, there exists some $\epsilon > 0$ and a C^1 function $\varphi : \,]-\epsilon, \epsilon[\longrightarrow \mathcal{U}$ which is a solution of the equation :

$$\frac{d\varphi}{dt} = f\,(t, \varphi(t)) \quad \forall t \;\in\,]-\epsilon, \epsilon[\tag{3.5}$$

with the initial condition :

$$\varphi(0) = \bar{x}$$

In particular, $\frac{d\varphi}{dt}(0) = f(0, \bar{x})$. If $f(0, \bar{x}) = 0$, the solution is trivial ($\varphi(t) = \bar{x}$ for all t); so we assume that $f(0, \bar{x})$ does not vanish.

Clearly, a differential equation on $\mathbb{R}$ is the simplest case of system of PDE on a manifold. Now, how do we solve this locally around 0 ? The intuition is to consider the *linearized* version of the differential equation. That is, we look for some linear function $\tilde{\varphi} = \alpha t + \beta$, which is a solution of the system that obtains when replacing in the right hand side the function $f\,(t, \varphi(t))$by its value at $t = 0$, namely $f(0, \bar{x})$. Such a solution obviously exists, and is given by :

$$\alpha = f(0, \bar{x}), \beta = \bar{x}$$

The question, at this point, is whether the existence of a solution for the linearized version of the problem implies the existence of a solution for the initial, non linear equation. Cauchy theorem essentially states that the answer is positive, under mild regularity conditions on f. The intuition, again, is pretty clear. Take some $\nu << \epsilon$, and contruct a linear approximation of the solution as follows :

- $\tilde{\varphi}(t) = f(0, \bar{x}).t + \bar{x}$ on $[0, \nu[$; then $\tilde{\varphi}(\nu) = f(0, \bar{x}).\nu + \bar{x}$ and from (3.5)

$$\frac{d\varphi}{dt}(\nu) = f\,(\nu, f(0, \bar{x}).\nu + \bar{x})$$

- then $\tilde{\varphi}(t) = \tilde{\varphi}(\nu) + f\,(\nu, f(0, \bar{x}).\nu + \bar{x})\,(t - \nu)$ on $[\nu, 2\nu[$ (that is, a linear continuation with slope $\frac{d\varphi}{dt}(\nu)$)
- and so on till $[\epsilon - \nu, \epsilon[$

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This piecewise linear function can be made arbitrarily close to the "true" solution when ν becomes arbitrarily small (and, again, under mild regularity conditions). This indicates why one may expect there should be a narrow link between existence problems in the linear and the nonlinear versions.

Can this result be generalized ? That is, can we expect that, for *any* system of PDE, the existence of a solution for its linearization guarantees the existence of a (local) solution to the general system ? Cartan-Kähler theorem provides a positive answer to this question. It requires, however, specific conditions to apply; the above statement would certainly not be true in full generality. To see why, consider the following two counter examples.

3.2.2. Counter example 1

We modify the previous example as follows. Take two functions f and g from $\mathbb{R}^2$ into $\mathbb{R}$, with $f(0,\bar{x}) = g(0,\bar{x}) = y \neq 0$ and consider the differential equation :

$$\frac{d\varphi}{dt} = f\,(t,\varphi(t)) \quad \forall t \ \in \,]-\epsilon,\epsilon\,[$$

$$\frac{d\varphi}{dt} = g\,(t,\varphi(t)) \quad \forall t \ \in \,]-\epsilon,\epsilon\,[$$

with the initial condition :

$$\varphi(0) = \bar{x}$$

Clearly, a solution does not exist in general (unless f and g coincide on some *open* neighbourhood of $(0,\bar{x})$). However, if we linearize at $(0,\bar{x})$, the linear version does have a solution, given by

$$\alpha = f(0,\bar{x}) = g(0,\bar{x}) = y, \ \beta = \bar{x}$$

So we are in a case where the linearized problem has a solution, but the latter does not locally extend. Why is that ? Clearly, the problem is that the equality $f(t,x) = g(t,x)$ - which is necessary for the existence of a solution - holds at $(0,\bar{x})$, but not in the neighbourhood; the technical translation being that $(0,\bar{x})$ is not an *ordinary point*. In general, what we need is a regularity condition which will exclude such pathological situations. Technically, this condition will state that the relevant equality holds true at ordinary points. Of course, this requires a general definition of the concept of an ordinary point. This is quite easy in the case above, but may be more complex in general. Cartan-Kähler theorem does precisely provide such a definition.

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3.2.3. Counter example 2

The first counter example introduced a system of differential equations. We now introduce several variables. Take two functions f and g from $\mathbb{R}^2$ into $\mathbb{R}$, and consider the system :

$$\begin{aligned} \frac{d\varphi}{dt_1} &= f(t_1, t_2) \\ \frac{d\varphi}{dt_2} &= g(t_1, t_2) \end{aligned} \tag{3.6}$$

with some initial condition (say, $\varphi(0,0) = 0$). As it is well known, a solution cannot exist unless f and g satisfy :

$$\frac{df}{dt_2} = \frac{dg}{dt_1}$$

However, if we linearize at $(0,0)$, the linear version does have a solution; the latter is given by $\bar{\varphi} = \alpha_1 t_1 + \alpha_2 t_2$, with

$$\alpha_1 = f(0,0), \alpha_2 = g(0,0)$$

We still are in a case where the linearized problem has a solution, but the latter does not locally extend. But the issue, here, is not whether the point is ordinary or not; all points are ordinary in this context. The problems rather comes from the fact that the system (3.6) is not "complete" (or "closed"), in the following sense. If these two equations are satisifed for all (t_1, t_2) in some open neighbourhood, then we can differentiate them; in particular, it must be the case that :

$$\begin{aligned} \frac{d^2\varphi}{dt_1 dt_2} &= \frac{\partial f(t_1, t_2)}{\partial t_2} \\ \frac{d^2\varphi}{dt_1 dt_2} &= \frac{\partial g(t_1, t_2)}{\partial t_1} \end{aligned} \tag{3.7}$$

These two equations are immediate ***analytic*** consequences of the previous ones. But, ***algebrically***, they are independent. In particular, the linearization of the system consisting of (3.6) ***and*** (3.7) is different from that of (3.6) alone.

The system (3.6) is incomplete (or, the language of EDC, "not closed") in the sense that is does not include some equations (such as (3.7) that are algebrically different from, but analytically implied by the initial ones. A second condition for applying Cartan-Kähler is that the system must be closed in the previous sense.

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3.2.4. The Cartan-Kähler theorem

A precise statement of the theorem is :

Theorem 3.1. *(Cartan-Kähler) Consider the exterior differential system :*

$$\omega^i = 0 , \quad 1 \leq i \leq K$$

Assume that the ω^k are real analytic and that they generate a differential ideal. Let $\bar{x}$ be a point and let $\bar{E}$ be a regular integral element at $\bar{x}$. Then there is a real analytic integral manifold M, containing $\bar{x}$ and such that :

$$T_{\bar{x}}M = \overline{E} .$$

Here, the differential ideal property is equivalent to 'completeness' in the sense illustrated above; an integral element $\bar{E}$ at $\bar{x}$ is a solution of the linearized system at $\bar{x}$, and it is regular if $\bar{x}$ is 'ordinary'; finally, $T_{\bar{x}}M$ is the tangent linear manifold to M at $\bar{x}$. For a precise mathematical definition of these terms, see Bryant et al. (1991).

3.3. The argument

We now come to the resolution of the market demand problem; again, the resolution of excess demand would obtain from a similar argument. An important remark, first, is that the system is complete, in the sense of the previous section. So the second condition of application of Cartan-Kähler is automatically fulfilled.

Following the approach alluded to above, the proof is in two steps.

- As a first step, one must look for a solution of the *linearized* problem (at some given point $\bar{\mathbf{p}}$). Specifically, choose (arbitrarily) the values at $\bar{\mathbf{p}}$ of λ_i and $\mathbf{\Delta}^i = D_{\mathbf{p}} V^i$ - say, $\bar{\lambda}_i$ and $\bar{\mathbf{\Delta}}^i$. In particular, one may choose $\bar{\lambda}_i < 0$, $\bar{\mathbf{\Delta}}^i << 0$ and $\bar{\mathbf{\Delta}} = (\bar{\mathbf{\Delta}}^1, ..., \bar{\mathbf{\Delta}}^n)$ invertible; if these properties hold at $\bar{\mathbf{p}}$, they will hold by continuity on a neighborhood as well. Also, these values must satisfy the relations defining the manifold $\mathcal{M}$:

$$\sum_i \bar{\lambda}_i \bar{\mathbf{\Delta}}^i = \mathbf{X}(\bar{\mathbf{p}})$$

and

$$\bar{\mathbf{p}}.\bar{\mathbf{\Delta}}^i = 1/\bar{\lambda}_i \quad \forall i$$

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- Now, linearize λ_i and $\mathbf{\Delta}^i$ (as functions of $\mathbf{p}$) around $\bar{\mathbf{p}}$:

$$\frac{\partial \lambda_i}{\partial p_j} = N_i^{\,j}$$
$$\frac{\partial \mathbf{\Delta}_k^i}{\partial p_j} = M_k^{i,j}$$

Solving the linearized problem is equivalent to finding vectors N^i and matrices M^i that satisfy the integration equations, i.e., (3.3), plus the equations expressing that λ_i and Δ^i remain on the manifold $\mathcal{M}$ (the latter obtain by differentiating (3.2)); in addition, we want the V^i to be convex.

Formally, we write that :

  - $\mathbf{\Delta}^i$ is the gradient of a convex function; this implies that

$$M^i \text{ symmetric positive, } i = 1, ..., n$$

  - 'the point remains on the manifold', which leads to :

$$D_{\mathbf{p}}\mathbf{X}(\bar{\mathbf{p}}) = \sum_i \left(\mathbf{\Delta}^i D_{\mathbf{p}}\lambda_i' + \lambda_i D_{\mathbf{p}}\mathbf{\Delta}^i\right) = \sum_i \left(\mathbf{\Delta}^i N_i' + \lambda_i M^i\right) \qquad (3.8)$$

$$M^i\mathbf{p} + \mathbf{\Delta}^i = -\frac{1}{\lambda_i^2} N_i \Leftrightarrow N_i' = -\lambda_i^2(\mathbf{p}'M^i + \mathbf{\Delta}^{i\prime}) \qquad (3.9)$$

  This is the set of linear equations that have to be solved in M^i and N_i.

- the second, and more tricky step is to show that the previous conditions hold true at *ordinary* points. This is crucial in order to go from a solution to the linearized version at each point to a solution to the general, non-linear problem; a move that may not be possible otherwise, as illustrated by the counter-examples in the previous section. Formally, this requirement translates into the fact that the subspaces involved have the 'right' codimension.

Is it possible to find vectors N^i and matrices M^i that satisfy the previous conditions ? The answer is positive; a general proof is in Appendix B. Two remarks can be made at this point :

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- the technique used in this proof applies not only to the aggregate demand and excess demand problem, but also to other problems of the same type. In particular, the case of incomplete markets is considered in Chiappori and Ekeland (1997b)
- in the case of market demand, the existence of a solution to the linearized problem (step one above) is in fact a consequence of known results of the literature, due to Sonnenschein (1973b), Diewert (1977) and Mantel (1977). These results, however, are not sufficient for the present purpose, because they do not allow to check the codimension properties of step two. The proof we provide does indeed allow to compute the required dimensions.

  To better understand the issue here, it is important to understand that finding *one* particular solution of the linearized problem cannot be enough. What we need, in fact, is a characterization of *all* possible solutions to the linearized problem - since we shall have to compute the dimensions of the corresponding spaces. This, of course, is more difficult than finding one solution. But it is a problem for which the whole apparatus of linear algebra can be used. This is what we do in Appendix B.

3.4. The main result

Once these conditions have been checked, Cartan-Kähler theorem applies. Finally, one gets the following statement :

Theorem 3.2. *Take $n \geq \ell$. Consider some open set $\mathcal{U}$ in $\mathbb{R}^\ell - \{0\}$ and some analytic mapping $\mathbf{X} : \mathcal{U} \to \mathbb{R}^n$ such that $\mathbf{p}.\mathbf{X}(\mathbf{p}) = n$. For all $\bar{\mathbf{p}} \in \mathcal{U}$ and for all $(\bar{\mathbf{x}}_1, ..., \bar{\mathbf{x}}_n) \in \mathbb{R}^{n^2}$ and $(\bar{\lambda}_1, ..., \bar{\lambda}_n) \in \mathbb{R}^n$ that satisfy :*

$$\bar{\mathbf{x}}_1 + ... + \bar{\mathbf{x}}_n = \mathbf{X}(\bar{\mathbf{p}})$$
$$\forall i, \quad \bar{\lambda}_i > 0$$

there exist n functions $U^1, ..., U^n$, where each U_i is defined in some convex neighborhood $\mathcal{U}_i$ of $\bar{\mathbf{x}}_i$ where it is analytic and strictly quasi-concave, n mappings $(\mathbf{x}_1, ..., \mathbf{x}_n)$ and n functions $(\lambda_1, ..., \lambda_n)$, all defined in some neighborhood $\mathcal{V}$ of $\bar{\mathbf{p}}$

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and analytic in $\mathcal{V}$, such that, for all $\mathbf{p} \in \mathcal{V}$:

$$\mathbf{p}.\mathbf{x}_i(\mathbf{p}) = 1, \quad i = 1, ..., n$$
$$U_i(\mathbf{x}_i(\mathbf{p})) = \max\{U_i(\mathbf{x}) \mid \mathbf{x} \in \mathcal{U}_i, \ \mathbf{p}.\mathbf{x} \leq 1\}, \quad i = 1, ..., n$$
$$\frac{\partial U_i}{\partial x^j}(\mathbf{x}_i(\mathbf{p})) = \lambda_i(\mathbf{p})\, p_j, \quad i = 1, ..., n, \quad j = 1, ..., n$$
$$\sum_{i=1}^{n} \mathbf{x}_i(\mathbf{p}) = \mathbf{X}(\mathbf{p})$$
$$\mathbf{x}_i(\bar{\mathbf{p}}) = \bar{\mathbf{x}}_i, \quad i = 1, ..., n$$
$$\lambda_i(\bar{\mathbf{p}}) = \bar{\lambda}_i, \quad i = 1, ..., n$$

Note that both the individual demands and the Lagrange multipliers (i.e., each agent's marginal utility of income) can be freely chosen at $\bar{\mathbf{p}}$. In particular, non-negativity constraints can be forgotten, since one can choose individual demands to be strictly positive at $\bar{\mathbf{p}}$, and they will remain positive in a neighbourhood.

4. Two extensions

We now consider two extensions of the latter result.

4.1. Continuity of the decomposition

The first extension is relative to what can be called the 'continuity' of the decomposition with respect to small perturbations of the initial function. Again, we present it in the market demand case, but it can be extended to excess demand in a straightforward manner.

Let $\mathbf{X}(\mathbf{p})$ be some given, analytic function. From the previous theorem, we know that, in a neighbourhood $\mathcal{U}$ of some given point $\bar{\mathbf{p}}$, $\mathbf{X}(\mathbf{p})$ can be decomposed as the sum of n individual market demands :

$$\mathbf{X}(\mathbf{p}) = \sum_{i=1}^{n} \mathbf{x}_i(\mathbf{p})$$

Let us fix a particular decomposition of this kind, and let ε be a positive scalar. Is it possible to find some positive ε' with the following property : for any analytic function $\mathbf{Y}(\mathbf{p})$ such that :

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$$\|\mathbf{X}(\bar{\mathbf{p}}) - \mathbf{Y}(\bar{\mathbf{p}})\| < \varepsilon'$$

it is possible to find, on some neighbourhood $\mathcal{U}' \subset \mathcal{U}$ of $\bar{\mathbf{p}}$, a decomposition of $\mathbf{Y}(\mathbf{p})$ as :

$$\mathbf{Y}(\mathbf{p}) = \sum_{i=1}^{n} \mathbf{y}_i(\mathbf{p})$$

such that :

- for all i, $\mathbf{y}_i(\mathbf{p})$ is an individual demand function, with $\mathbf{p}.\mathbf{y}_i(\mathbf{p}) = 1$
- $\|\mathbf{x}_i(\mathbf{p}) - \mathbf{y}_i(\mathbf{p})\| < \varepsilon \qquad \text{for all } \mathbf{p} \in \mathcal{U}$

In words, the decomposition described in the main theorem must be 'robust' to small perturbations of the initial function, in the sense that any function 'close to' the initial one can be decomposed into individual demands that are 'close to' the initial ones.

The answer is positive, and is in fact an immediate consequence of the technique we adopt here. Indeed, remember that the value, at $\bar{\mathbf{p}}$, of individual demands can be chosen arbitrarily (provided they add up to $\mathbf{X}(\bar{\mathbf{p}})$). So we can choose the $\mathbf{y}_i(\bar{\mathbf{p}})$ such that :

$$\mathbf{y}_i(\bar{\mathbf{p}}) = \mathbf{x}_i(\bar{\mathbf{p}}) - \frac{\mathbf{X}(\bar{\mathbf{p}}) - \mathbf{Y}(\bar{\mathbf{p}})}{n}$$

which implies that

$$\|\mathbf{y}_i(\bar{\mathbf{p}}) - \mathbf{x}_i(\bar{\mathbf{p}})\| < \frac{\varepsilon'}{n} \qquad (4.1)$$

Also, both the $\mathbf{x}_i(\mathbf{p})$ and the $\mathbf{y}_i(\mathbf{p})$ are analytic on $\mathcal{U}$, so that, for any ε, one can always find some ε' such that (4.1) implies

$$\|\mathbf{x}_i(\mathbf{p}) - \mathbf{y}_i(\mathbf{p})\| < \varepsilon \qquad \text{for all } \mathbf{p} \in \mathcal{U}$$

In fact, one can even do (slightly) better, thanks to the following remark. Assume that, at $\bar{\mathbf{p}}$, $\mathbf{Y}$ is close to $\mathbf{X}$ in the C^1 sense; i.e., we also have that :

$$\|D_{\mathbf{p}}\mathbf{X}(\bar{\mathbf{p}}) - D_{\mathbf{p}}\mathbf{Y}(\bar{\mathbf{p}})\| < \varepsilon'$$

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Now, consider the basic equations of the linearized version, i.e., (3.8) and (3.9). Replacing $\mathbf{X}$ by $\mathbf{Y}$ does not change the latter, while in the former the left hand side is modified only by ε'. We know, then, that whichever solution that has been chosen to construct the M^i matrices (i.e, the partials of $\mathbf{x}_i(\mathbf{p})$) can be approximated to construct the partials of $\mathbf{y}_i(\mathbf{p})$. It follows that the $\mathbf{y}_i(\mathbf{p})$ can be chosen close to the $\mathbf{x}_i(\mathbf{p})$ in the C^1 sense as well; i.e., such that :

$$\left\| D_{\mathbf{p}}\mathbf{x}_i(\mathbf{p}) - \mathbf{D}_{\mathbf{p}}\mathbf{y}_i(\mathbf{p}) \right\| < \varepsilon \qquad \text{for all } \mathbf{p} \in \mathcal{U}$$

Finally, the same technique applies to the excess demand problem, hence leads to a similar robustness conclusion (note, however, that the result is only local).

4.2. Arbitrary income distributions

4.2.1. Fixed income distributions

The second extension is specific to the market demand case. Theorem 3.2 has been established under the assumtion that each member's income was constant, and equal to 1 :

$$\mathbf{p}.\mathbf{x}_i(\mathbf{p}) = 1, \quad i = 1, ..., n$$

A natural question is whether it extends to more general income distributions. In fact, we consider the following, general version of the problem. Take some arbitrary functions $\mu_1(\mathbf{p}), ..., \mu_n(\mathbf{p})$ that satisfy $\mu_i(\bar{\mathbf{p}}) > 0$, $i = 1, ..., n$, and $\sum \mu_i(\mathbf{p}) = n$ for all $\mathbf{p}$. Here, $\mu_i(\mathbf{p})$ is interpreted as i's nominal income, which is allowed to depend on $\mathbf{p}$ in an arbitrary way. Now, can we locally (around $\bar{\mathbf{p}}$) decompose some arbitrary, analytic function $\mathbf{X}(\mathbf{p})$ as the sum of n individual demands $\mathbf{x}_i(\mathbf{p})$, such that each $\mathbf{x}_i(\mathbf{p})$ is a solution to the program :

$$\begin{aligned} &\max U^i(\mathbf{x}^i) \\ &\quad \mathbf{p}.\mathbf{x}^i = \mu_i(\mathbf{p}) \end{aligned}$$

for some well-chosen utility functions ?

Again, the answer is positive, at least for analytic distributions, as stated in the following theorem :

Theorem 4.1. *Take $n \geq \ell$. Consider some open set $\mathcal{U}$ in $\mathbb{R}^n - \{0\}$, some analytic functions $\mu_1(\mathbf{p}), ..., \mu_n(\mathbf{p})$ that satisfy $\mu_i(\bar{\mathbf{p}}) > 0$, $i = 1, ..., n$, and $\sum \mu_i(\mathbf{p}) = n$ for*

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all $\mathbf{p}$, *and some analytic mapping* $\mathbf{X} : \mathcal{U} \to \mathbb{R}^n$ *such that* $\mathbf{p}.\mathbf{X}(\mathbf{p}) = 1$. *For all* $\bar{\mathbf{p}} \in \mathcal{U}$ *and for all* $(\bar{\mathbf{x}}_1, ..., \bar{\mathbf{x}}_n) \in \mathbb{R}^{n^2}$ *and* $(\bar{\lambda}_1, ..., \bar{\lambda}_n) \in \mathbb{R}^n$ *that satisfy :*

$$\bar{\mathbf{x}}_1 + ... + \bar{\mathbf{x}}_n = \mathbf{X}(\bar{\mathbf{p}})$$
$$\forall i, \quad \lambda_i > 0$$

there exist n *functions* $U^1, ..., U^n$, *where each* U_i *is defined in some convex neighborhood* $\mathcal{U}_i$ *of* $\bar{\mathbf{x}}_i$ *where it is analytic and strictly quasi-concave,* n *mappings* $(\mathbf{x}_1, ..., \mathbf{x}_n)$ *and* n *functions* $(\lambda_1, ..., \lambda_n)$, *all defined in some neighborhood* $\mathcal{V}$ *of* $\bar{\mathbf{p}}$ *and analytic in* $\mathcal{V}$, *such that, for all* $\mathbf{p} \in \mathcal{V}$ *:*

$$\begin{aligned}
\mathbf{p}.\mathbf{x}_i(\mathbf{p}) &= \mu_i(\mathbf{p}), \quad i = 1, ..., n \\
U_i(\mathbf{x}_i(\mathbf{p})) &= \max \{U_i(\mathbf{x}) \mid \mathbf{x} \in \mathcal{U}_i, \ \mathbf{p}.\mathbf{x} \leq \mu_i(\mathbf{p})\}, \quad i = 1, ..., n \\
\frac{\partial U_i}{\partial x^j}(\mathbf{x}_i(\mathbf{p})) &= \lambda_i(\mathbf{p})\, p_j, \quad i = 1, ..., n, \ \ j = 1, ..., n \\
\sum_{i=1}^{n} \mathbf{x}_i(\mathbf{p}) &= \mathbf{X}(\mathbf{p}) \\
\mathbf{x}_i(\bar{\mathbf{p}}) &= \bar{\mathbf{x}}_i, \quad i = 1, ..., n \\
\lambda_i(\bar{\mathbf{p}}) &= \bar{\lambda}_i, \quad i = 1, ..., n
\end{aligned}$$

The proof is in Appendix C; in fact, it is exactly identical to the previous one, since the $\mu_i(\mathbf{p})$ only introduce minor changes into the basic argument. Note, in particular, that this encompasses the case where $\mathbf{p}.\mathbf{x}_i(\mathbf{p}) = \mu_i$ for some fixed income distribution $\mu_1, ..., \mu_n$ that satisfies $\sum \mu_i = n$; this is the way the problem is often stated in the literature. This result in the market demand case is the counterpart of the fact that, in the excess demand case, the distribution of initial endowments can be chosen arbitrarily.

4.2.2. Unconstrained income distributions

Finally, consider the following, quite different problem. Assume that, instead of being initially given, the $\mu_i(\mathbf{p})$ can be arbitrarily chosen to "match" the initial function $\mathbf{X}$. Economically, the interpretation is that there exist some price-dependent income distribution, upon which we have no information. Now, how many individual agents are necessary to decompose the market demand $\mathbf{X}$ in this new setting?

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In fact, the answer to this question is not completely known yet. But two related results can be given. One is the answer to a simpler problem, namely the 'mathematical integration' viewpoint. That is, we shall in what follows disregard convexity requirements; we simply impose that $\mathbf{X}(\mathbf{p})$ be written as a linear composition of k gradients, where k denotes the number of consumers:

$$\mathbf{X}(\mathbf{p}) = \lambda_1(\mathbf{p})\, D_{\mathbf{p}}V^1(\mathbf{p}) + ... + \lambda_k(\mathbf{p})\, D_{\mathbf{p}}V^k(\mathbf{p})$$

Now, in this case, the answer can readily be derived from some well known result in EDC, namely Pfaff theorem. It can be showed that only $n/2$ consumers are necessary to solve the 'mathematical integration' problem (see Chiappori and Ekeland (1996) for a precise proof).

A second conclusion stems from a recent result by Zakalsukin, who extends Pfaff theorem to show the following : if, in addition, the functions V^i are required to be convex, then a solution can always be found with one more consumer - i.e., in this case, $n/2 + 1$. Hence, a interesting conclusion is that, when income distribution can freely vary, then $n/2 + 1$ consumers at most are sufficient to generate any (acceptable) excess demand.

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APPENDIX

A. Proof of Theorem 3.2

We proove the result for $n = \ell$. We are looking for an n-dimensional integral manifold of the exterior differential system :

$$\sum_j d\Delta_j^i \wedge dp^j = 0 \quad \forall i \leq n$$

satisfying

$$dp^1 \wedge \ldots \wedge dp^n \neq 0$$

in the n^2-dimensional submanifold $\mathcal{M}$ of $\mathbb{R}^{2n+n^2}$ defined by:

$$\mathcal{M} = \{(p^i, \lambda_j, \Delta_l^k) \mid \mathbf{X}_i(\mathbf{p}) = \sum_j \lambda_j \Delta_i^j \text{ and } \bar{\mathbf{p}}'.\overline{\mathbf{\Delta}^i} = 1/\overline{\lambda}_i \quad \forall i\}$$

Note first that this system is obviously closed in the sense of Cartan (all exterior derivatives of the equations in the system vanish identically)

A.1. The linearized problem

We first consider the resolution of the linearized problem described in Section 3.3. Consider some given point $\bar{\mathbf{p}}$, and choose (arbitrarily) the values of $\overline{\lambda}_i$ and $\overline{\mathbf{\Delta}^i} = (\overline{\mathbf{\Delta}_1}^i, \ldots, \overline{\mathbf{\Delta}_n}^i)$ of λ_i and $\mathbf{\Delta}^i$ at $\bar{\mathbf{p}}$ so that $\lambda_i < 0$, all components of $\mathbf{\Delta}^i$ are negative , $\mathbf{\Delta} = (\mathbf{\Delta}^1, \ldots, \mathbf{\Delta}^n)$ is invertible, and:

$$\begin{aligned} \mathbf{X}(\bar{\mathbf{p}}) &= \sum_i \lambda_i \mathbf{\Delta}^i \\ \bar{\mathbf{p}}'.\overline{\mathbf{\Delta}^i} &= 1/\overline{\lambda}_i \quad \forall i \end{aligned}$$

Now, we linearize λ_i and $\mathbf{\Delta}^i$ (as functions of $\mathbf{p}$) around $\bar{\mathbf{p}}$:

$$\begin{aligned} \frac{\partial \lambda_i}{\partial p^j} &= N_{i,j} \\ \frac{\partial \mathbf{\Delta}_k^i}{\partial p^j} &= M_{k,j}^i \end{aligned}$$

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We are thus looking for n vectors N_i and n matrices M^i such that :

$$M^i \text{ is symmetric positive definite, } i = 1, ..., n$$

and (differentiating the equations (3.2) which define the manifold $\mathcal{M}$):

$$D_{\mathbf{p}}\mathbf{X}(\overline{\mathbf{p}}) = \sum_i \left(\overline{\mathbf{\Delta}}^i D_{\mathbf{p}}\lambda_i' + \overline{\lambda}_i D_{\mathbf{p}}\mathbf{\Delta}^i\right) = \sum_i \left(\overline{\mathbf{\Delta}}^i N_i' + \overline{\lambda}_i M^i\right) \quad \text{(A.1)}$$

$$M^i\overline{\mathbf{p}} + \overline{\mathbf{\Delta}}^i = -\frac{1}{\overline{\lambda}_i^2} N_i \Leftrightarrow N_i' = -\overline{\lambda}_i^2(\overline{\mathbf{p}}' M^i + \overline{\mathbf{\Delta}}^{i\prime}) \quad \text{(A.2)}$$

From now on, to ease the notations, we will drop the overbar, and write $\mathbf{p}^i$, $\mathbf{\Delta}_j^i$, λ_i instead of $\overline{\mathbf{p}}^i$, $\overline{\mathbf{\Delta}}_j^i$, $\overline{\lambda}_i$. But it is understood that these values are fixed as above.

- Substituting (A.2) into (A.1) gives :

$$S + \sum_i \lambda_i^2 \mathbf{\Delta}^i \mathbf{\Delta}^{i\prime} = \sum_i \lambda_i M^i - \sum_i \lambda_i^2 \mathbf{\Delta}^i \mathbf{p}' M^i \quad \text{(A.3)}$$

where $S = D_{\mathbf{p}}\mathbf{X}(\bar{\mathbf{p}})$ is given and is such that (by differentiating $\mathbf{p}'\mathbf{X} = 1$):

$$\mathbf{p}'S = -\mathbf{X}'$$

.It implies that:

$$\mathbf{p}'\left(S + \sum_i \lambda_i^2 \mathbf{\Delta}^i \mathbf{\Delta}^{i\prime}\right) = -\mathbf{X}' + \sum_i \lambda_i \mathbf{\Delta}^{i\prime} = 0$$

We now concentrate upon the set of all solutions to (A.3). In our case, it should be considered as an equation on the set of symmetric, positive definite matrices. However, for reasons that will become clear in the computation of Cartan characters, we shall also consider (A.3) as an equation on the set of all $(n \times n)$ matrices.

As a consequence, let us first study the two operators Φ and Φ_S defined by:

$$\Phi : \left(\mathbb{R}^{n^2}\right)^n = \{\left(M^1, ..., M^n\right) \mid M^i \in \mathbb{R}^{n^2}\} \to \mathbb{A}$$

$$\Phi\left(M^1, ..., M^n\right) = \sum_i \lambda_i M^i - \sum_i \lambda_i^2 \mathbf{\Delta}^i \mathbf{p}' M^i \quad \text{(A.4)}$$

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where **A** the set of $(n \times n)$ matrices such that $\mathbf{p}'A = 0$, and

$$\Phi_S : \mathbb{S}^n = \{\left(M^1, ..., M^n\right) \mid M^i \in \mathbb{S}\}$$

$$\Phi_S\left(M^1, ..., M^n\right) = \sum_i \lambda_i M^i - \sum_i \lambda_i^2 \mathbf{\Delta}^i \mathbf{p}' M^i \tag{A.5}$$

where $\mathbb{S}$ is the set of $(n \times n)$ symmetric matrices. Note that Φ_S is simply the restriction of Φ to $\mathbb{S}^n$

We wish to prove that Φ and Φ_S are onto. To do this, we first characterize the kernel of Φ_S. This is done as follows:

Lemma 1. *$(M^1, ..., M^n)$ belong to $\ker \Phi_S$ if and only if there exists a symmetric, $(n \times n)$ matrix $\beta = ((\beta^{k,s}))$ such that*

$$M^k \mathbf{p} = \sum_s \lambda_s^2 \beta^{k,s} \mathbf{\Delta}^s \tag{A.6}$$

$$\sum_k \lambda_k M^k = \sum_{k,s} \lambda_s^2 \lambda_k^2 \beta^{k,s} \mathbf{\Delta}^k (\mathbf{\Delta}^s)' \tag{A.7}$$

Proof. Let $(M^1, ..., M^n) \in \ker \Phi_S$. We have

$$\sum_i \lambda_i M^i_{k,j} - \sum_{il} \lambda_i^2 \mathbf{\Delta}^i_k \mathbf{p}^l M^i_{lj} = 0. \tag{A.8}$$

Write that the antisymmetric part vanishes:

$$\sum_{il} \lambda_i^2 (\mathbf{\Delta}^i_k M^i_{l,j} \mathbf{p}^l - \mathbf{\Delta}^i_j M^i_{l,k} \mathbf{p}^l) = 0$$

which can be rewritten, using exterior products:

$$\sum_i (\lambda_i^2 \Delta^i) \wedge (M^i \mathbf{p}) = 0$$

By a celebrated Lemma of Elie Cartan (see Cartan 1945), this is equivalent to the existence of some symmetric matrix β such that :

$$M^i \mathbf{p} = \sum_j \lambda_j^2 \mathbf{\Delta}^j \beta^{i,j}$$

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Reporting in (A.8) leads to :

$$\sum_k \lambda_k M^k = \sum_{k,s} \lambda_s^2 \lambda_k^2 \beta^{k,s} \mathbf{\Delta}^k (\mathbf{\Delta}^s)'$$

The two previous relations fully characterize the kernel of Φ_S.

■

In particular, we must check that these two equations are compatible. Indeed, we have that :

$$\sum_k \lambda_k M^k \mathbf{p} = \sum_s \lambda_k \lambda_s^2 \beta^{k,s} \mathbf{\Delta}^s$$

from (A.6), and

$$\sum_k \lambda_k M^k \mathbf{p} = \sum_{k,s} \lambda_s^2 \lambda_k^2 \beta^{k,s} \mathbf{\Delta}^k (\mathbf{\Delta}^s)' \mathbf{p}$$

from (A.7). But since $(\mathbf{\Delta}^s)'\mathbf{p} = 1/\lambda_s$, and β is symmetric, these relations are equivalent.

We now investigate the range of Φ_S . Let L_1 denote the subspace generated by the right hand sides of (A.6) and (A.7):

$$L_1 = \left\{ (\mathbf{w}_1, ..., \mathbf{w}_n, W) \in \mathbb{R}^{n^2} \times \mathbb{S} \mid \exists \beta : \sum_s \lambda_s^2 \mathbf{\Delta}^s \beta_{i,s} = \mathbf{w}_i \text{ and } \sum_{k,s} \lambda_s^2 \lambda_k^2 \beta^{k,s} \mathbf{\Delta}^k (\mathbf{\Delta}^s)' = W \right\}$$

It can readily be checked that the operator $\beta \rightarrow (\mathbf{w}_1, ..., \mathbf{w}_n, W)$ is injective; it follows that

$$\dim L_1 = \frac{n(n+1)}{2}$$

Then

$$\ker \Phi_S = \left\{ \left(M^1, ..., M^n\right) \in \mathbb{S}^n \mid \left(M^1\mathbf{p}, ..., M^n\mathbf{p}, \sum_k \lambda_k M^k\right) \in L_1 \right\}$$

Consider the operator $G_S : \mathbb{S}^n \rightarrow \mathbb{R}^{n^2} \times \mathbb{S}$ defined by:

$$G_S\left(M^1, ..., M^n\right) = \left(M^1\mathbf{p}, ..., M^n\mathbf{p}, \sum_k \lambda_k M^k\right)$$

. Its image belongs to the subspace L_2 defined by :

$$L_2 = \left\{ (\mathbf{w}_1, ..., \mathbf{w}_n, W) \in \mathbb{R}^{n^2} \times \mathbb{S} \mid W\mathbf{p} = \sum_k \lambda_k \mathbf{w}_k \right\}$$

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By the compatibility condition iust checked,

$$L_1 \subset L_2$$

Hence

$$\begin{aligned} \dim \ker \Phi_S &= \dim G_S^{-1}(L_1) \\ &= \dim L_1 + \dim \ker G_S \\ &= \dim L_1 + \dim \mathbb{S}^n - \dim L_2 \end{aligned}$$

So that:

$$\begin{aligned} \dim \operatorname{Im} \Phi_S &= \dim L_2 - \dim L_1 \\ &= n^2 - n \end{aligned}$$

But Φ_S maps $\mathbb{S}^n$ to $\mathbb{A}$, with $\dim \mathbb{A} = n^2 - n$. It follows that Φ_S, considered as a linear mapping from $\mathbb{S}^n$ to $\mathbb{A}$, is onto. Of course, Φ is then onto as well. All these results can be summarized as follows :

Lemma 2. *For any matrix S such that $\mathbf{p}'S = -\mathbf{X}'$, there exist n symmetric matrices $(M^1, ..., M^n)$ such that*

$$S + \sum_i \lambda_i^2 \mathbf{\Delta}^i \mathbf{\Delta}^{i\prime} = \sum_i \bar{\lambda}_i M^i - \sum_i \lambda_i^2 \mathbf{\Delta}^i \mathbf{p}' M^i \tag{A.9}$$

Moreover, the set of all such n-uples is an affine space of dimension $\frac{1}{2}n^2(n+1) - n(n-1)$.

The case of Φ is exactly similar, except for the initial space, which is $\left(\mathbb{R}^{n^2}\right)^n$ (hence of dimension n^3) instead of $\mathbb{S}^n$ (hence of dimension $\frac{1}{2}n^2(n+1)$). The corresponding result writes :

Lemma 3. *For any matrix S such that $\mathbf{p}'S = -\mathbf{X}'$, there exist n matrices $(M^1, ..., M^n)$ such that*

$$S + \sum_i \lambda_i^2 \mathbf{\Delta}^i \mathbf{\Delta}^{i\prime} = \sum_i \bar{\lambda}_i M^i - \sum_i \lambda_i^2 \mathbf{\Delta}^i \mathbf{p}' M^i \tag{A.10}$$

Moreover, the set of all such n-uples is an affine space of dimension $n^3 - n(n-1)$

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These two results, and particularly the dimensions of the kernels, will be used in the computation of Cartan characters in the next subsection.

The final step is to show that there exist a solution to the previous equation such that all matrices $(M^1, ..., M^n)$ are *positive* definite. The key idea is to show the following Lemma :

Lemma 4. *There exist n symmetric, positive definite matrices $Q^1, ..., Q^n$ such that :*

$$\left(Q^1, ..., Q^n\right) \in \ker \Phi$$

Indeed, assume this Lemma holds true, and let $(M^1, ..., M^n)$ be any n-uple of symmetric matrices such that (A.10) is satisfied. Then for any positive scalar k, the n-uple $(M^1 + kQ^1, ..., M^n + kQ^n)$ satisfies (A.10); moreover, for k large enough, these matrices are positive definite, which would close the proof.

We now prove Lemma 4. We know from Lemma 1 that, to any n-uple of symmetric matrices $(Q^1, ..., Q^n)$ in the kernel, one can associate a symmetric matrix β satisfying the relations (A.6) and (A.7). For the sake of simplicity, define P^k and $\gamma_{k,s}$ by :

$$P^k = \lambda_k Q^k, \qquad \gamma_{k,s} = \lambda_s^2 \lambda_k^2 \beta^{k,s}$$

The P^i must be negative (remember $\lambda_k < 0$), and satisfy the equations :

$$P^k \mathbf{p} = \sum_s \frac{\gamma_{k,s}}{\lambda_k} \mathbf{\Delta}^s = \Delta\Gamma \frac{\mathbf{e}_k}{\lambda_k}$$

$$\sum_k P^k = \sum_{k,s} \gamma_{k,s} \mathbf{\Delta}^k (\mathbf{\Delta}^s)' = \Delta\Gamma\Delta'$$

where $\mathbf{e}_k' = (0, ..., 1, ...0)$ is the kth vector of the canonical basis, Γ denotes the matrix $(\gamma_{k,s})$, and Δ denotes the matrix $(\mathbf{\Delta}^1, ..., \mathbf{\Delta}^n)$.

Using the fact that the matrix Δ is invertible, we may define Γ^k by :

$$P^k = \Delta\Gamma^k\Delta'$$

Note that there is a one-to-one correspondence between the P^k and the Γ^k, and that, in addition, P^k is symmetric (resp. negative definite) if and only if Γ^k is symmetric (resp. negative definite). Also, we have that $\mathbf{p}.\mathbf{\Delta}^k = 1/\lambda_k$ for all

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k, which implies that $\Delta'\mathbf{p} = \sum_i \frac{\mathbf{e}_i}{\lambda_i}$. So we are looking for n matrices Γ^k that are symmetric, negative definite, and such that :

$$\Gamma^k \left(\sum_i \frac{\mathbf{e}_i}{\lambda_i}\right) = \left(\sum_i \Gamma^i\right) \frac{\mathbf{e}_k}{\lambda_k}$$

(then $\Gamma = \sum_i \Gamma^i$).

The next remark is that we may suppose $\lambda_k = 1\ \forall k$. Indeed, assume that the Γ^k are solutions of the previous problem with $\lambda_k = 1$, then define $\bar{\Gamma}^k$ by $\bar{\gamma}^k_{i,j} = \lambda_i \lambda_j \gamma^k_{i,j}$; then the $\bar{\Gamma}^k$ are solutions of the initial problem. We are thus facing the problem :

$$\Gamma^k \left(\sum_i e_i\right) = \left(\sum_i \Gamma^i\right) e_k \tag{A.11}$$

to be solved by symmetric, negative definite matrices Γ^k.

We can now exhibit a particular solution $\Gamma^{(1)} = \left(\Gamma^{(1)1}, \ldots, \Gamma^{(1)n}\right)$ by :

$$\Gamma^{(1)1} = \begin{pmatrix} \gamma_1^1 & \gamma_2^1 & \gamma_3^1 & \cdots & \gamma_n^1 \\ \gamma_2^1 & \gamma_2^1 & \gamma_3^1 & \cdots & \gamma_n^1 \\ \gamma_3^1 & \gamma_3^1 & \gamma_3^1 & \cdots & \gamma_n^1 \\ \cdots & \cdots & \cdots & \cdots & .. \\ \gamma_n^1 & \gamma_n^1 & \gamma_n^1 & \cdots & \gamma_n^1 \end{pmatrix}$$

$$\Gamma^{(1)2} = \begin{pmatrix} \gamma_2^1 & \gamma_2^1 & \gamma_3^1 & \cdots & \gamma_n^1 \\ \gamma_2^1 & \gamma_2^1 & \gamma_3^1 & \cdots & \gamma_n^1 \\ \gamma_3^1 & \gamma_3^1 & \gamma_3^1 & \cdots & \gamma_n^1 \\ \cdots & \cdots & \cdots & \cdots & .. \\ \gamma_n^1 & \gamma_n^1 & \gamma_n^1 & \cdots & \gamma_n^1 \end{pmatrix}$$

....

$$\Gamma^{(1)n} = \begin{pmatrix} \gamma_n^1 & \gamma_n^1 & \gamma_n^1 & \cdots & \gamma_n^1 \\ \gamma_n^1 & \gamma_n^1 & \gamma_n^1 & \cdots & \gamma_n^1 \\ \gamma_n^1 & \gamma_n^1 & \gamma_n^1 & \cdots & \gamma_n^1 \\ \cdots & \cdots & \cdots & \cdots & .. \\ \gamma_n^1 & \gamma_n^1 & \gamma_n^1 & \cdots & \gamma_n^1 \end{pmatrix}$$

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These matrices are symmetric and satisfy (A.11); moreover, one can easily fix the coefficients such that they all are negative ($\gamma_i < \gamma_{i+1} < 0$). The only problem left is that, among them, only $\Gamma^{(1)1}$ is definite. Thus the final step is to define other solutions using permutations permutation of rows and columns; i.e., we define $\Gamma^{(i)} = \left(\Gamma^{(i)1}, ..., \Gamma^{(i)n}\right)$ by

$$\Gamma^{(i)1} = \begin{pmatrix} \gamma_1^i & \cdots & \gamma_1^i & \gamma_{i+1}^i & \cdots & \gamma_n^i \\ \cdot\cdot & \cdots & \cdots & \cdot\cdot & \cdots & \cdots \\ \gamma_1^i & \cdot\cdot & \gamma_1^i & \gamma_{i+1}^i & \cdots & \gamma_n^i \\ \gamma_{i+1}^i & \cdots & \gamma_{i+1}^i & \gamma_{i+1}^i & \cdots & \gamma_n^i \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdot\cdot \\ \gamma_n^i & \cdots & \gamma_n^i & \gamma_n^i & \cdots & \gamma_n^i \end{pmatrix}$$

$$\Gamma^{(i)i} = \begin{pmatrix} \gamma_i^i & \gamma_i^i & \cdots & \gamma_i^i & \gamma_{i+1}^i & \cdots & \gamma_n^i \\ \gamma_i^i & \gamma_{i-1}^i & \cdots & \gamma_{i-1}^i & \gamma_{i+1}^i & \cdots & \gamma_n^i \\ \cdot\cdot & \cdot\cdot & \cdots & \cdots & \cdot\cdot & \cdots & \cdots \\ \gamma_i^i & \gamma_{i-1}^i & \cdot\cdot & \gamma_1^i & \gamma_{i+1}^i & \cdots & \gamma_n^i \\ \gamma_{i+1}^i & \gamma_{i+1}^i & \cdots & \gamma_{i+1}^i & \gamma_{i+1}^i & \cdots & \gamma_n^i \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdot\cdot \\ \gamma_n^i & \gamma_n^i & \cdots & \gamma_n^i & \gamma_n^i & \cdots & \gamma_n^i \end{pmatrix}$$

$$\Gamma^{(i)n} = \begin{pmatrix} \gamma_n^i & \gamma_n^i & \gamma_n^i & \cdots & \gamma_n^i \\ \gamma_n^i & \gamma_n^i & \gamma_n^i & \cdots & \gamma_n^i \\ \gamma_n^i & \gamma_n^i & \gamma_n^i & \cdots & \gamma_n^i \\ \cdots & \cdots & \cdots & \cdots & \cdot\cdot \\ \gamma_n^i & \gamma_n^i & \gamma_n^i & \cdots & \gamma_n^i \end{pmatrix}$$

These matrices have the same properties as the $\Gamma^{(1)k}$, except that here only $\Gamma^{(i)i}$ is definite. Finally, take the Γ^k defined by :

$$\Gamma^k = \sum_i a_i \Gamma^{(i)\,k}, \quad a_i > 0$$

These matrices are symmetric, negative definite, and solve the equation above, QED ■

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A.2. Computing the Cartan characters

We now proceed to the second step of our proof.

A first task is to compute the codimension of G^n (the set of all n -dimensional integral elements; we refer to [4], [5] and [6] for this computation) in the Grassmannian manifold of all tangent n -planes . To do this, take $x = (\mathbf{p}, \lambda, \boldsymbol{\Delta}) \in \mathcal{M}$. Any n-dimensional subspace E of the tangent space $T_x\mathcal{M}$ satisfying (??) is defined by a set of equations :

$$\begin{aligned} d\lambda_i &= \sum_j N_{ij} dp^j \\ d\Delta_k^i &= \sum_j M_{kj}^i dp^j \end{aligned}$$

where the N_{ij} and the M_{kj}^i satisfy (A.1) and (A.2). The map $E \to \left(N_{ij}, M_{kj}^i\right)$ is well-defined and one-to-one, so that the $\left(N_{ij}, M_{kj}^i\right)$ provide a local coordinate system for the set of n-dimensional subspaces of $T_x\mathcal{M}$. Also, Lemma 3 above shows that the set of $\left(N_{ij}, M_{kj}^i\right)$ that satisfy (A.1) and (A.2) is a subspace of dimension $n^3 - n(n-1)$. This is precisely the dimension of $\mathbb{R}^n \times \mathbb{P}^n(\mathbb{R}^n)$, as expected.

We now have to find the codimension of G^n in the Grassmannian $\mathbb{R}^n \times \mathbb{P}^n(\mathbb{R}^n)$. In the $\left(N_{ij}, M_{kj}^i\right)$ coordinate system, G^n is defined by (A.1) and (A.2) plus the additional equations $M_{kj}^i = M_k^{ji}$. Lemma ?? above shows that the set of solutions to these has dimension $\frac{1}{2}n^2(n+1) - n(n-1)$. The codimension of G^n is thus :

$$\begin{aligned} c &= \left[n^3 - n(n-1)\right] - \left[\frac{1}{2}n^2(n+1) - n(n-1)\right] \\ &= \frac{1}{2}n^2(n-1) \end{aligned}$$

Now, fix $\bar{x} = (\bar{\mathbf{p}}, \bar{\lambda}, \bar{\boldsymbol{\Delta}}) \in \mathcal{M}$, with $\bar{\lambda} << \mathbf{0}$. From the previous subsection, we know that we can find N_{ij} and M_{kj}^i such that the matrices $\bar{M}_k$ are symmetric and positive definite. Let $\bar{E}$ the corresponding integral element; we claim that it is ordinary. Indeed, consider the 1-forms :

$$\begin{aligned} \pi_k^i &= d\Delta_k^i - \sum_j \bar{M}_{kj}^i dp^j \\ \pi^i &= \sum_j \bar{N}_{ij} dp^j \end{aligned}$$

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so that $\bar{E}$ is defined by $\pi_k^i = 0, \pi^i = 0$. We have :

$$\sum_k d\Delta_k^i \wedge dp^k = \sum_k \left(\pi_k^i - \sum_j \bar{M}_{kj}^i dp^j\right) \wedge dp^k = \sum_k \pi_k^i \wedge dp^k$$

since $\bar{M}_k$ is symmetric. By a standard criterion, we have that, for $0 \leq \ell \leq n$:

$$H_\ell^* = Span\left\{\pi_k^i \mid i \leq n, k \leq \ell\right\}$$

Therefore $c_\ell = \dim H_\ell^* = \ell n$ for $\ell \leq n$, and :

$$c_0 + ... + c_{n-1} = n\,(0 + 1 + ... + (n-1)) = n^2\frac{n-1}{2}$$

This coincides with c, the codimension of G^n. So $(\bar{x}, \bar{E})$ is ordinary and we can apply the Cartan-Kähler theorem.

B. Proof of Theorem 3.2

It obtains by slightly modifying the previous proof. Indeed, the problem at $\bar{\mathbf{p}}$ becomes :

$$\mathbf{X}(\bar{\mathbf{p}}) = \sum_i \lambda_i \mathbf{\Delta}^i$$

$$\bar{\mathbf{p}}'.\mathbf{\Delta}^i = \frac{\mu_i(\bar{\mathbf{p}})}{\lambda_i} \quad \forall i$$

hence equation (A.1) is unchanged, while (A.2) becomes :

$$M^i\mathbf{p} + \mathbf{\Delta}^i = -\frac{\mu_i(\bar{\mathbf{p}})}{\lambda_i^2}N_{ij} + \frac{D_{\mathbf{p}}\mu_i}{\lambda_i} \Leftrightarrow N_i' = -\frac{\lambda_i^2}{\mu_i(\bar{\mathbf{p}})}(\mathbf{p}'M^i + \mathbf{\Delta}^{i\prime}) + \frac{\lambda_i}{\mu_i(\bar{\mathbf{p}})}(D_{\mathbf{p}}\mu_i)' \tag{B.1}$$

Substituting (B.1) into (A.1) gives :

$$S + \sum_i \frac{\lambda_i^2}{\mu_i(\bar{\mathbf{p}})}\mathbf{\Delta}^i\mathbf{\Delta}^{i\prime} - \sum_i \frac{\lambda_i}{\mu_i(\bar{\mathbf{p}})}\mathbf{\Delta}^i (D_{\mathbf{p}}\mu_i)' = \sum_i \bar{\lambda}_i M^i - \sum_i \frac{\lambda_i^2}{\mu_i(\bar{\mathbf{p}})}\mathbf{\Delta}^i\mathbf{p}'M^i \tag{B.2}$$

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Note that :

$$\mathbf{p}'\sum_i \frac{\lambda_i}{\mu_i(\bar{\mathbf{p}})}\boldsymbol{\Delta}^i (D_{\mathbf{p}}\mu_i)' = \sum_i (D_{\mathbf{p}}\mu_i)' = 0 \quad \text{since } \sum_i \mu_i = n$$

so that we still have that

$$\mathbf{p}'\left(S + \sum_i \lambda_i^2 \boldsymbol{\Delta}^i \boldsymbol{\Delta}^{i\prime} - \sum_i \frac{\lambda_i}{\mu_i(\bar{\mathbf{p}})}\boldsymbol{\Delta}^i (D_{\mathbf{p}}\mu_i)'\right) = 0$$

Now, the proof does not depend on the left hand side of (B.2). As for the right hand side, we may define :

$$\bar{\boldsymbol{\Delta}}^i = \frac{\boldsymbol{\Delta}^i}{\mu_i(\bar{\mathbf{p}})}$$

so that

$$\bar{\mathbf{p}}'.\bar{\boldsymbol{\Delta}}^i = \frac{1}{\lambda_i} \quad \forall i$$

Then the the right hand side of (B.2) becomes

$$\sum_i \bar{\lambda}_i M^i - \sum_i \lambda_i^2 \bar{\boldsymbol{\Delta}}^i \mathbf{p}' M^i$$

and the previous proof exactly applies.

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